

Codimension one and two bifurcations for the FitzHugh-Nagumo system

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ABSTRACT

In this paper results of a theoretical and numerical investigation of the bifurcations [3] associated with the FitzHugh–Nagumo system [1] when two parameters are varied are presented. For a two-dimensional continuous dynamical system we revealed an unexpectedly large number of bifurcations: 7 different types of codimension one and 18 different types of codimension two bifurcations. Owing to them there are more than 60 types of topologically non-equivalent dynamics. The results are synthesized in the global bifurcation diagram.

Some of the bifurcation boundaries (such as those corresponding to Hopf or saddle–node bifurcations) are found analytically, for others (corresponding to homoclinic bifurcation [5] and to non-hyperbolic limit cycle bifurcation) asymptotical approximations are found using the normal form method [4]. Other bifurcation boundaries (such as those corresponding to the bifurcation that breaks the connection between saddles) are found numerically.

The ranges of the parameter plane corresponding to one, two or three limit cycles are found [2]. The range corresponding to limit cycles surrounding a concave domain and the range where the canard phenomenon is present are also emphasized. An analogy between the bifurcations which are present around a point of double homoclinic bifurcation and around a point of double breaking saddle connection bifurcation is given.

References

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Keywords: *bifurcation, FitzHugh–Nagumo, canard phenomenon, global bifurcation diagram*

Mathematics Subject Classification: *34A47*

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