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Transformation of matrix families to versal deformations

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ABSTRACT

In the paper a family of matrices $A(p)$ smoothly (or analytically) dependent on a vector of parameters p is considered. The versal deformation theory for matrices developed by Arnold (1971) gives systematic way to analyze the matrix family and its eigenvalues in the neighborhood of a point in the parameter space. Versal deformations (normal forms) are matrix families such that the smooth change of basis and parameters induce locally any matrix family with the same initial matrix. It allows studying different properties of parameter dependent systems of ordinary differential equations, singularities of bifurcation diagrams, stability boundaries etc. Both qualitative and quantitative analysis is possible. For qualitative analysis (like classification of singularities) we need to know only the form of a versal deformation. Versal deformations for matrices of different types, like real, complex, Hamiltonian, reversible and other matrices were studied and listed in a number of papers. Applications of versal deformations for quantitative study of matrix families (analysis of stability boundaries, eigenvalue behavior etc.) needs to know the appropriate change of basis (which is a function of parameters) and the change of parameters. The problem of finding these transformation functions was solved only for the case, when matrices belong to the space of all real or complex matrices, see Mailybaev (2000).

In this paper a general method of finding transformation functions for matrix families of arbitrary type is given. The transformation functions are found as Taylor series, where coefficients are obtained from explicit recurrent procedure. In particular, the form of the procedure is given for the cases, when the matrices are elements of classical Lie algebras with involution. Application of these results to singularity and stability theory is discussed.

References:

V.I. Arnold (1971) On matrices depending on parameters, Russian Math. Surveys, 26, 29–43.

A.A. Mailybaev (2000) Transformation of families of matrices to normal forms and its application to stability theory, SIAM J. Matr. Anal. Appl., 21 (2), 396–417.

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