Section 08: Ordinary Differential Equations and Dynamical Systems | Poster number 45

Canonical forms of singularities of second order differential equations and blow up

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ABSTRACT

Motivated by questions in observable dynamics [4] we consider planar vector fields of the form $X(x_1, x_2) = x_2 \partial/\partial x_1 + f(x_1, x_2)\partial/\partial x_2$, which correspond to a second order differential equation, and study canonical forms with respect to changes of variables of the form $(y_1, y_2) = \phi(x_1, x_2) = (g(x_1), g'(x_1)x_2)$. Such a transformation ϕ respects the structure of 'being a second order equation'. We study these subjects near a singularity at the origin. According to the properties of f we conjugate X to a 'simplified' form, which is, under some reasonable assumptions, unique and is a modulus of stability, i.e. two equations are conjugate iff their canonical form is the same.

First the rather trivial case: if $f_0 = f(0, 0) \neq 0$ then for each $\tilde{f}_0 \neq 0$ the canonical form is $y_2 \partial/\partial y_1 + (\tilde{f}_0 + f_1(y_1, y_2)y_2)\partial/\partial y_2$.

The more interesting case is when $f(0, 0) = 0$. Especially when $\partial f/\partial x_1(0, 0) = 0$ the desingularization technique known as blow up permit us to show and explain why the canonical form can, generically, only be obtained by a finitely smooth change of variables. More precisely: suppose there is a smallest positive integer $n \geq 2$ such that $b := \partial^n f/\partial x_1^n(0, 0) \neq 0$. Then the canonical form $y_2\partial/\partial y_1 + ((by_1^n/n!) + \tilde{\gamma}(y_1, y_2)y_2)\partial/\partial y_2$ can be obtained by a C^{n-1} transformation ϕ which is in general *not* of class C^n. The main ideas are the following. Denote $S(v) = f(v, 0) - bv^n/n!$. We then study the degenerate singularity at the origin of the vector field $Z(u, v) = bu^n/n!\partial/\partial u + ((bv^n/n!) + S(v))\partial/\partial v$. This is done efficiently using the blow up desingularization technique [1, 3]. After that we apply a Sternberg type normal form theorem. The desired transformation ϕ is then constructed from a well selected integral curve of Z.

References

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