

Splitting of separatrices associated to parabolic points

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ABSTRACT

When we perturb a two dimensional system having separatrices by a time periodic perturbation, generically the connexion breaks down and arises what is called splitting of separatrices.

An standard way to measure the magnitude of the splitting is by calculating the area of a lobe generated the invariant manifolds by using the Poincaré-Melnikov method.

In case of rapidly forced perturbations, the straightforward application of the Poincaré-Melnikov method has not been justified in general. Several works deals with this question when the fixed point is hyperbolic.

The aim of this work is to give an asymptotic formula of the area of the lobe that remains between the two invariant curves, in the case that the origin is a parabolic fixed point. We consider Hamiltonian systems with Hamiltonian $h(x, y, t/\epsilon) = y^2/2 + V(x) + \mu\epsilon^p h_1(x, y, t/\epsilon)$, where $V(x)$ is a polynomial of order three at zero and $h_1(x, y, t/\epsilon)$ is a polynomial in x, y variables of order greater or equal than the order of $V(x)$ and it is a periodic function with respect to t/ϵ with zero averaging.

The proof has five parts: existence of local invariant manifolds, extension theorem of invariant manifolds, existence of flow box canonical coordinates, existence of transversal homoclinics points and effective computation of area. The most significant contribution is the proof of the existence of flow box coordinates using only that the manifold can be expressed as a graph, near the origin, which is the case when the fixed point is parabolic.

References

- [1] A. Delshams, M.T. Seara. Splitting of separatrices in Hamiltonian systems with $1\frac{1}{2}$ degrees of freedom. *Math. Phys. Electron. J.*, **3** (1997), 1–40.
- [2] E. Fontich. Stable curves asymptotic to degenerate fixed point. *Nonlinear Analysis* bf 35 (1999) 711–733.

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