

On the existence of an invariant torus close to a quasi-torus of an exact symplectic map

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ABSTRACT

Many physical problems can be studied using mathematical models that are perturbations of one that can be solved completely.

In some cases though, it turns out that the actual size of the perturbation is not small enough as it is required by the classical KAM theory. However, it is well-known that it is still possible to numerically compute accurate approximations to invariant tori (quasi-tori) outside the range of applicability of the KAM theorem. Then, it seems natural to ask for a KAM theory in which the perturbative parameter is not the size of the perturbation but the accuracy of the computation. The method presented in this work is intended to be an initial step in this direction.

More concretely, let f be an analytic exact symplectic map of \mathbb{R}^{2n} ($n \geq 1$), having an approximate invariant torus corresponding to a given (and known) Diophantine frequency vector. Under a generic non-degeneracy condition we show that, if the initial approximation is good enough, there exists a true invariant torus close to the approximate one, with the same frequency vector. The distance between the true torus and the initial approximation is of the order of the accuracy of the quasi-invariant torus.

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