3rd European Congress of Mathematics
Poster sessions

Section 08: Ordinary Differential Equations and Dynamical Systems | Poster number 678

Stability of certain planar unbounded polycycles

A. Gasull, Universitat Autònoma de Barcelona.
V. Mañosa*, Universitat Politècnica de Catalunya.
F. Mañosas, Universitat Autònoma de Barcelona.

ABSTRACT

Graphics with hyperbolic corners are one of the simplest graphics considered in the literature. In fact, already Poincaré introduced the so called *graphic number,* γ which in most cases decides the stability of the graphic. Γ is stable (respectively unstable) if $\gamma > 1$ (respectively $\gamma < 1$.) The above result also can be stated saying that the return map of a transversal section Σ, trough a regular point of Γ has a return map of the form

$$\Pi(x) = \Delta x^{\gamma} + o(x^{\lambda}), \tag{1}$$

The main goal of this paper is to give an effective way of studying this stability when $\gamma = 1$. We are able to compute Δ in formula (1), for some concrete graphics appearing in the Poincaré compactification of some families of polynomial differential equations that we describe in the sequel. This value is obtained in terms of improper convergent integrals. Note that when $\Delta \neq 1$, by using (1) the stability of Γ can be obtained from this value.

One of the families considered is the so called polynomial Kolmogorov systems, and includes several differential equations studied in ecological models. It can be written as

$$\begin{aligned} \dot{x} &= x\, f(x,y)\,, \\ \dot{y} &= y\, g(x,y)\,, \end{aligned}$$

where f and g are also polynomials. The graphic that we consider is the one formed by the two positive semi-axes and the equator of the Poincaré sphere. The value Δ is given. We also present a result which guarantees the bifurcation of one limit cycle from this graphic. This limit cycle can be interpreted as a big amplitude oscillation for the Kolmogorov system. Other families are studied.

Keywords: *Polynomial vector field, stability,polycycle, limit cycle*

Mathematics Subject Classification: *58F10, 34C23*

Contact Address: `victor.manosa@upc.es`