Section 09: Partial Differential Equations | Poster number 129

The nonexistence theorems for solutions of capillary problem in the absence of gravity.

A. A. Kosmodem'yanskii, Dept. Comp. Math., Moscow.

ABSTRACT

Let D_0 and D_1 $\quad (D_0 \subset D_1)$ be plane convex figures. Denote by A_i and p_i the area and the perimeter of figure $D_i \quad (i = 0, 1)$.

In the absence of gravity, the equation of liquid free surface takes the form

$$div\, Tu = \frac{p_1 \cos\gamma}{A_1}, \tag{1}$$

in domain D_1 with boundary condition

$$(Tu, n) = \cos\gamma, \qquad (Tu = \nabla u/\sqrt{1 + |\nabla u|^2}), \tag{2}$$

where n is the outward normal and $0 \leq \gamma \leq \pi/2$.

If $\gamma = 0$ the important condition of the existence of solution for this problem in the domain D_1 is the following : let D_0 be an arbitrary subdomain of D_1 then if the solution of the problem (1) – (2) exists then inequality

$$\frac{A_1}{p_1} > \frac{A_0}{p_0} \tag{3}$$

holds ([1]). Using the notion of mixed square ([2] we can reformulate this result as sufficient condition of nonexistence of solution for problem (1) – (2). Let A_{01} be a mixed square of domains D_0 and D_1.

Theorem 1. Let there exists a convex subdomain D_0 of domain D_1 such that the inequality

$$(p_0 + p_1)A_1 < 2A_{01}p_1.$$

holds. Then if $\gamma = 0$ then a solution of (1) – (2) does not exist.

Using theorem 1 we obtain

Theorem 2. Let we can inscribe into D_1 the disk of radius

$$r > \frac{p_1 A_1}{p_1^2 - 2\pi A_1}.$$

Then the problem (1) – (2) has no solutions for any contact angle γ satisfying the inequality

$$\cos\gamma > \frac{A_1}{rp_1}\Big(1 + \sqrt{\frac{4\pi(p_1 r - \pi r^2 - A_1)}{p_1^2 - 4\pi A_1}}\Big).$$

References

[1] Concus P., Finn R. On capillary free surfaces in the absence of gravity. Acta Math. 132 (1974), pp. 177–198.

[2] W. Blaschke, Kreis und Kugel, Walter de Gruyter & Co, Berlin (1956).

Keywords: *Capillary Problem, Nonexistence*

Mathematics Subject Classification: *35J25, 35C15*

Contact Address: `mitja@farber.mccme.ru`