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On Initial-Boundary Value Problems for Systems not of Cauchy-Kovalevskaya Type

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ABSTRACT

We consider a class of integrodifferential systems

$$\mathcal{L}(D_t, D_t^{-1}, D_x)u = f(t,x), \quad x = (x_1, \dots, x_n),$$

where $D_t^{-1}u(t,x) = \int\limits_0^t u(s,x)\,ds$.

1. The operator $\mathcal{L}(D_t, D_t^{-1}, D_x)$ has the form

$$\mathcal{L}(D_t, D_t^{-1}, D_x) = \begin{pmatrix} K_0 \circ D_t + K_1 + K_2 \circ D_t^{-1} & L(D_x) \\ M(D_x) & N(D_x) \circ D_t^{-1} \end{pmatrix},$$

where K_0, K_1, K_2 are $(m \times m)$ numerical matrices, $\det K_0 \neq 0$, $L(D_x)$, $M(D_x)$, $N(D_x)$ are $m \times (\nu - m)$, $(\nu - m) \times m$, $(\nu - m) \times (\nu - m)$ matrix differential operators respectively.

2. The symbol $\mathcal{L}(\tau, \tau^{-1}, i\xi)$ satisfies the homogeneity condition

$$\mathcal{L}(\tau, \tau^{-1}, ic\xi) = S(c)\mathcal{L}(\tau, \tau^{-1}, i\xi)T(c), \quad c > 0,$$

where $S(c) = \{c^{\mathrm{s}_i}\delta_{ij}\}$, $T(c) = \{c^{\mathrm{t}_j}\delta_{ij}\}$, $\mathrm{s}_i \leq 0$, $\mathrm{t}_j \geq 0$ are integers, $\max_{1\leq i\leq\nu} \mathrm{s}_i = 0$, $\mathrm{s}_k \geq \mathrm{s}_1$, $\mathrm{t}_k \geq \mathrm{t}_1$, $k = m+1, \dots, \nu$, δ_{ij} are Kronecker's delta symbols.

3. The exists a number $\gamma_0 > 0$ such that $\det \mathcal{L}(\tau, \tau^{-1}, i\xi) = 0$ for $\mathrm{Re}\,\tau \geq \gamma_0$, $\xi \in R_n$, if and only if $\xi = 0$.

The considerable class of systems contains the Sobolev system, the system of internal waves and the system of gravitational gyroscopical waves for the Boussinesq approximation. The Cauchy problem for the given class of systems was studied in [1]. In the present paper we consider initial-boundary value problems in the quadrant $R_{n+1}^{++} = \{(t,x):\ t > 0,\ x' = (x_1, \dots, x_{n-1}),\ x_n > 0\}$. We establish results on solvability in the weighted Sobolev spaces $W_{p,\gamma,\sigma}^{l_1,l_2}(R_{n+1}^{++})$, $1 < p < \infty$, $\gamma > 0$, $\sigma \geq 0$, with the norm

$$\|u(t,x), W_{p,\gamma,\sigma}^{l_1,l_2}(R_{n+1}^{++})\| = \|(1+|x|)^{-\sigma}e^{-\gamma t}D_t^{l_1}u(t,x), L_p(R_{n+1}^{++})\|$$

$$+ \sum_{|\alpha|\leq l_2} \|(1+|x|)^{-\sigma(1-|\alpha|/l_2)}e^{-\gamma t}D_x^{\alpha}u(t,x), L_p(R_{n+1}^{++})\|.$$

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Reference

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