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Asymptotic behaviour of degenerate nonlinear Dirichlet problems in perforated domains of general structure

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ABSTRACT

The asymptotic behaviour of the solutions of degenerate nonlinear second order elliptic equations with Dirichlet boundary conditions in perforated domains of general structure is studied provided that the weight function belongs to a certain Muckenhoupt class.

Let Ω be an arbitrary bounded domain in the n-dimensional Euclidean space $\mathbf{R^n}$ and let $\Omega^{(s)} \subset \Omega, s = 1, 2, \dots$, be a sequence of subdomains. In $\Omega^{(s)}$ we consider a nonlinear elliptic boundary value problem

$$\sum_{j=1}^{n} \frac{d}{dx_j} a_j \left(x, u, \frac{\partial u}{\partial x} \right) = a_0 \left(x, u, \frac{\partial u}{\partial x} \right), \qquad x \in \Omega^{(s)}, \tag{1}$$

$$u(x) = f(x), \qquad x \in \partial\Omega^{(s)}. \tag{2}$$

Our conditions on the data of problem (1), (2) provide the existence of a solution $u_s(x) \in f(x) + \overset{\circ}{\rightarrow} W_p^1\left(\Omega^{(s)}, w\right)$ of problem (1), (2) for every s and also the boundedness of the sequence $u_s(x)$ in $W_p^1\left(\Omega^{(s)}, w\right)$, where $w(x)$ belongs to the certain Muckenhoupt class.

We don't assume any geometric conditions over domains $\Omega^{(s)}$. Our hypothesis about the structure of the sets $\Omega \backslash \Omega^{(s)}$ is the following: there exists a sequence r_s tending to zero such that the inequality $cap_{p,w}\left(K(x_0, r) \backslash \Omega^{(s)}\right) \leq w\left(K(x_0, r)\right)$ is valid for every $x_0 \in \Omega$ and every $r \geq r_s$, where $cap_{p,w}\left(K(x_0, r) \backslash \Omega^{(s)}\right), w\left(K(x_0, r)\right)$ are weighted capacity and measure of the corresponding sets, and $K(x_0, r)$ denotes the cube with the centre x_0 and edge $2r$.

We establish the strong convergence in $W_m^1(\Omega, w), m < p$, of solutions $u_s(x)$ of problems under consideration and construct a boundary value problem satisfied by the limit function. We construct also a corrector for the approximation of $u_s(x)$ in $W_p^1(\Omega, w)$. All our results are based on the asymptotic expansion of the sequence $u_s(x)$ and on a pointwise estimate of the solution of a model degenerate nonlinear Dirichlet problem.

Keywords: *homogenization, perforated domain, degenerate nonlinear elliptic equation, Muckenhoupt class, asymptotic behaviour*

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