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For the homogeneous Navier-Stokes equations, every finite Dirichlet integral assumes its global maximum value at the initial moment

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ABSTRACT

A new approach to the well-known nonlinear initial boundary value problem

$$\begin{cases} v_t + (v,\nabla)v - \nu\Delta v + \nabla p = 0,\ \operatorname{div} v = 0,\ (x,t) \in Q_T = \Omega \times (0,T), \\ v\big|_{t=0} = v^0(x),\ v\big|_{\partial\Omega} = 0,\ \operatorname{div} v^0 = 0,\ v^0\big|_{\partial\Omega} = 0, \end{cases} \tag{1}$$

in a bounded domain $\Omega \subset \Re^3$ with $\partial\Omega \in C^2$, has made it possible to estimate a Dirichlet integral and establish that its global maximum over $t \in [0,T]$ is equal to the initial value of the integral, with no restriction imposed on the initial value. This extends the result previously available only for the small initial values of a Dirichlet integral — to all positive initial values. The boundedness of a Dirichlet integral is known to be necessary and sufficient for the existence of a global solution

$$\{v, \nabla p\} \in W_{2,x,t}^{2,1}(Q_T; \Re^3) \times L^2(Q_T; \Re^3), \tag{2}$$

usually referred to as strong, where an anisotropic Sobolev space is involved, which consists of all vector functions $v(x,t)\colon Q_T \to \Re^3$ with the square-summable weak derivatives of the second order in x and of the first order in t. Solution of the class (2) requires the initial data from a Sobolev space $W_2^1(\Omega; \Re^3)$ consisting of all $\Re^3$-valued vector functions with the square-summable first order weak derivatives. Given any positive ν and T, for any $v^0 \in W_2^1(\Omega; \Re^3)$ satisfying the compatibility conditions contained in (1), there exists a unique solution (2) of problem (1), and

$$\max_{t\in[0,T]} \sum_{i,j=1}^{3} \int_\Omega \left(\frac{\partial v_i}{\partial x_j}\right)^2 dx = \sum_{i,j=1}^{3} \int_\Omega \left(\frac{\partial v_i^0}{\partial x_j}\right)^2 dx.$$

By Serrin's unicity theorem, the existence of a global strong solution (2) implies the uniqueness of Hopf's global weak solution. Hence, Hopf's global weak solution proves to be a strong solution of the class (2), being even classically smooth should v^0 and $\partial\Omega$ be sufficiently smooth. In the latter case, the classical smoothness readily follows from Solonnikov's estimates. The approach is based on solving first a certain auxiliary problem, employing the structure of nonlinearity $(v,\nabla)v$.

Keywords: *Navier-Stokes equations, Dirichlet integral, global existence, global estimates*

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