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Black swans and canards

E. Shchepakina, Samara State University, Samara, Russia.

ABSTRACT

Under modeling of self–ignition regime in the case of an autocatalytic combustion reaction an unexpected and very interesting fact was discovered: a canard appeared as a basic model. The term "canard" or "French duck" is comparatively recent in scientific literature. It has been introduced by French mathematicians investigating van der Pol's equation in conformity to cycles.

Consider the following singularly perturbed system

$$\dot{x} = f(x, y, z, \varepsilon), \tag{1}$$

$$\dot{y} = g(x, y, z, \varepsilon), \tag{2}$$

$$\epsilon \dot{z} = p(x, y, z, \alpha(y, \epsilon), \varepsilon) \tag{3}$$

where ε is a small positive parameter, α is a scalar continuous function, x and z are scalar variables, y is a vector of dimension n.
The *slow surface* S of system (1)–(3) is the surface described by the equation

$$p(x, y, z, \alpha(y, 0), 0) = 0. \tag{4}$$

Let $z = \phi(x, y)$ be an isolated solution of equation (4). We call the subset S^s_ϕ (S^u_ϕ) of S defined by $\frac{\partial p}{\partial z}(x, y, \phi(x, y), \alpha(y, 0), 0) < 0$ (> 0) the attracting (repelling) subset of S.

The subset of S defined by $\frac{\partial p}{\partial z}(x, y, \phi(x, y), \alpha(y, 0), 0) = 0$ is called the *breakdown surface*. Its dimension is equal to $\dim y$.

In ϵ–neighborhood of S^s_ϕ (S^u_ϕ) there exists a attracting (repelling) slow integral manifold. The slow integral manifold is defined as a smooth invariant surface of slow motions.
The availability of the additional scalar function $\alpha(y, \epsilon)$ provides the possibility of gluing attracting and repelling slow integral manifolds along the breakdown surface. As a result we get a slow integral surface of variable stability (or black swans). The existence theorem and properties of black swans are obtained. This surface is considered as natural generalization of the notion of a canard. Recall that a trajectory of a singularly perturbed system of differential equations is called a canard, if it follows at first a attracting integral manifold, and then a repelling one. Thus, the black swan is transmuted into canard under $\dim y = 0$.

It should be noted that applications of canards and black swans in chemical kinetics problems are obtained in the paper.

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Contact Address: `elenash@ssu.samara.ru`