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Integral geometry problems for matrices and reconstruction problems for operators in a vector bundle

Vertgeim Lev Borisovich, Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences.

ABSTRACT

I. 1. $D \subset R^2$, $\Gamma = \partial D$, $\Gamma \in C^1$. $x = \xi(l), y = \eta(l), l \in [0, L]$ — natural parametrization of Γ. K — a regular family of curves in $\bar{D}$. $P \in C^1(\bar{\Omega}, M_{nn}), Q \in C^1(\bar{\Omega}, M_{mm})$, $\Omega = D \times [0, L]$.

Problem 1. Determine $U \in C^1(\bar{D}, M_{nm})$ by the known

$$\int_{K(l_1,l_2)} P(x, y, l_1)\, U(x, y)\, Q(x, y, l_1)\, ds = V(l_1, l_2),\ l_1, l_2 \in [0, L].$$

Theorem 1. *If* $\det P, \det Q \neq 0$ *everywhere and* $P_l P^{-1}, Q^{-1} Q_l$ *— sufficiently small, then the problem* 1 *has no more than one solution and*

$$\int\int_D \sum_{ij} u_{ij}^2(x, y)\, dxdy \le C \int_0^L \int_0^L \mathrm{tr}(\frac{\partial V^t}{\partial l_1} \frac{\partial V}{\partial l_2})\, dl_1 dl_2.$$

A similar scalar problem was solved in [1].

2. $A \in C^2(\bar{D}, M_{nn})$. $x = x(t), y = y(t)$; $x(0) = \xi(l_1), y(0) = \eta(l_1)$ — a natural parametrization of a curve from K with the length $T(l_1, l_2)$.

$$\frac{dU}{dt} = A(x(t), y(t))\, U,\ U(0) = E.\ F(l_1, l_2) = U(T(l_1, l_2)).$$

Problem 2. Determine A in $\bar{D}$ by the known F on $[0, L] \times [0, L]$.
Let $A \in M = M(M_1, M_2, M_3) \subset C^2(\bar{D}, M_{nn})$ (defined by some upper estimates for A and ∇A).

Theorem 2. *If* M_j *are sufficiently small, then the problem* 2 *has in* M *no more than one solution and for* $A_1, A_2 \in M$ *and corresponding* F_1, F_2

$$\int\int_D \sum_{ij} (a_{1,ij} - a_{2,ij})^2 dxdy \le \tilde{C} \int_0^L \int_0^L \mathrm{tr}(\frac{\partial R^t}{\partial l_1} \frac{\partial R}{\partial l_2}) dl_1 dl_2,\ \ R = F_1^{-1} F_2 - E.$$

II. $(M, \partial M, \tilde{g})$ — Riemannian manifold; (η, g) — Hermitian vector bundle over M. $A(x), P(x, \xi), Q(x, \xi)$ — linear operators, acting in the fibers η_x, $x \in M$, $(x, \xi) \in TM$. An integral transform of A along geodesics with the weights P, Q is defined, a uniqueness and stability theorem is proven for the corresponding integral geometry problem. A nonlinear problem for A along geodesics has been studied. Some methods of [2] has been used.

References

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Contact Address: `vert@math.nsc.ru`