

**Involutive and  $C^*$ -Complexifications: Commutative Real and Complex Involutive Complete Algebras in Effective Perspective**

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**ABSTRACT**

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The natural setting for the original complexification forms that provided by the backwards assignment of the real numbers to the complex ones and not of the underlying real, two-dimensional, vector space to  $\mathbb{C}$ . This subtle distinction leads to the consideration of the refined notions of an involutive and a  $C^*$ -complexification for commutative real complete normed, or multinormed, algebras, in which the existence of the involution constitutes the prominent feature. In a categorical vein, the accomplished functors now establish certain equivalences of appropriate categories, a privilege that enables us not only to present a comparison theory for a thorough study of the real and complex involutive algebras involved, but also to derive new results. More specifically: (i) faithful dense representations of the algebras quoted as function algebras on their spectra proper are obtained in both cases, followed by the classical Stone-Gel'fand duality; (ii) new insights into the structure of commutative  $C^*$ -algebras are revealed, by pointing out that they are constructed from abstract real function algebras ( $\mathbb{R}$ -algebras) precisely with the same pattern as this of the complex field from the real one, namely, the  $C^*$ -complexification; (iii) a constructive realization for the enveloping  $C^*$ -algebra of an involutive Banach algebra is available with illustrative applications to group and disc algebras; (iv)  $\mathbb{R}$ -algebras are intrinsically characterized by equivalent conditions including one of partial ordering; and in conclusion, (v) several open questions regarding the structure of real Banach algebras are answered.

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