

On the theory of a generalized Hilbert boundary value problem

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ABSTRACT

We consider the following problem. Find a function $\varphi^+(z)$ analytic in $|z| < 1$ and satisfying the condition

$$\operatorname{Re}\{A(t)\varphi^+(t) + B(t)\varphi^+(\alpha(t))\} = h(t) \text{ on } |t| = 1 \quad (1)$$

where A, B, h are continuous functions on $|t| < 1$, $\alpha(t)$ is a diffeomorphism of $|t| = 1$ onto itself preserving or changing the orientation on $|t| = 1$.

The Fredholm theory of problem (1) is constructed.

If $\alpha(t)$ is a linear fractional diffeomorphism on $|t| = 1$ satisfying the Carleman condition $\alpha(\alpha(t)) \equiv t$, then the number of linear independent solutions and solvability conditions of problem (1) are calculated, and the very solutions are constructed in the terms of a factorization of (2×2) -matrix function determined by coefficients A, B . The particular cases are analysed when this factorization is realized effectively.

Keywords: *Shift operator, Fredholm theory, singular integral equations, factorization of matrix functions*

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