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**On the definition of approximation for isometrical semigroupsassociated with the cocycle conjugacy on the CAR algebra**

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**ABSTRACT**

Given a linear operator $R,\ 0 < R < I$, in a Hilbert space $H$, it is introduced the notion of the $R$-approximation for two one-parameter isometrical semigroups $V_T$ and $U_T$ in $H$ satisfying the property $V_t R = RV_t,\ U_t R = RV_t,\ t \in T$, where $T = \mathbf{Z}_+$ or $\mathbf{R}_+$. The semigroups $V_T$ and $U_T$ are called to be $R$-approximating each other if they have minimal dilations $V'_T$ and $U'_T$ in a Hilbert space $H',\ H \subset H'$, such that $R^{1/2}(V'_t - U'_t) \in s_2,\ V_t^{'*}|_{H' \ominus H} = U_t^{'*}|_{H' \ominus H},\ t \in T$. The operator $R$ defines the quasifree state $\omega_R$ on the $C^*$-algebra of the canonical anticommutation relations (CAR) $A(H)$ over $H$ by the formula $\omega_R(a^*(f)a(g)) = (g, Rf)_H,\ f, g \in H$, where $a^*(f), a(g)$ are the creation and the annihilation operators in the antisymmetric Fock space $F(H)$. The algebra $A(H)$ is generated by $a^*(f), a(g),\ f, g \in H$, and coincides with the algebra of all bounded operators $B(F(H))$. In the GNS (Gelfand - Naimark - Segal) representation $\pi_R$ associated with $\omega_R$, $A(H)$ generates the hyperfinite factor $M_R = \pi_R(A(H))$ that can be of type I,II or III. One can lift $V_T$ and $U_T$ up to one-parameter endomorphisms semigroups $B_R(V_T)$ and $B_R(U_T)$ on the factor $M_R$. In this case the condition of R-approximation is saficient for the cocycle conjugacy of $B_R(V_T)$ and $B_R(U_T)$. Put $R_t = P_H V_t^{'*}|_H,\ t \in T$, where $P_H$ is a projection on $H$. Then the pair $(V_T^{'*}, R_T)$ defines the quasifree quantum stochastic process $j$ on $M_R$. The R-approximation of $V_T$ is associated with the perturbation $j$ by a Markovian cocycle. The asymptotic properties arising under such perturbation of the flow of Powers shifts is discussed.

**Reference**

G.G. Amosov, On approximation of continuous semigroups of isometries in a Hilbert space, Izvest. Vysch. Uchebn. Zaved. Matem. [Russ. Math.] 2 (2000) 3–9.

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