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Solvability Of Some Operator-Differential Equations In Complex Plane And Related To It Some Spectral Properties

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ABSTRACT

1. Consider the next problem

$$\frac{dU}{dz} = \sum_{i,j=0}^{\infty} A_{ij} U^i z^j, \tag{1}$$

$$U(0) = 0, \tag{2}$$

where U is a desired linear bounded operator, acting in Banach space X, $A_{ij} : X \to X$ are given linear operators, provided that
$\sum\limits_{i,j=0}^{\infty} \|A_{i,j}\| a^i b^j \leq M < \infty$, where $a, M = const$, z- complex variable, $|z| \leq b$, $b = const$, $\sup\limits_{|z|\leq b} \|U(z)\| \leq a$.
Investigation of such problems in scalar case began from the works by Cauchy, Painlevé, Riccati and others (for detail, see f.e.[1]). Generalization of these results to the infinite dimensional case presents interest.
The solution of the problem (1), (2) is sought in the form
$U(z) = \sum\limits_{k=0}^{\infty} B_k z^k$ in some domain $|z| < r$,where $U(z) \in L(X)$ and $L(X)$ is a Banach algebra of linear bounded operators acting in X.

Theorem: The problem (1), (2) has unique holomorf solution in the domain $|z| < r_1$, where
$r_1 = b_1 \left(1 - \exp\left(-\frac{a_1}{2b_1 M}\right)\right)$, $r_{1,}$ $a_1$and $b_1$are constants such that $r_1 < r$, $a_1 < a$, $b_1 < b$.

2. Now consider the next spectral problem:

$$z\frac{dU}{dz} - zA_{10} - z^2 A_{20} - zA_{11}U - A_{02}U^2 - ... = \lambda U \tag{3}$$

$$U(0) = 0 \tag{4}$$

in the space $L(X)$. Here λ is a complex parameter, and $A_{ij} \in L(X)$. Assume that the series in (3) converges absolutely in the sence of X in some neighbourhood of the point $z = 0$, $|z| < r$,and $\sup\limits_{|z|<r} \|U(z)\| < R$,
$R = const$.
Such problems, in scalar case, were investigated by Briot and Bonquet [1]. In spite of that, their results were interesting, they didn't take into consideration the spectral questions related to this problem.

Theorem: All of natural numbers are eigenvalues of the problem (3), (4). Otherwise, there is a unique holomorf solution of this problem for every $\lambda = n$ $(n = 1, 2, ...)$.

Remark: It would be noted that, in the equation (3) the coefficient $A_{01} = 0$. The case $A_{01} \neq 0$ requires more deep analysis. The linear operator–differential equations in complex domain were investigated by E.Hille, M.G.Kreyn, U.L.Daletski, A.M.Akhmedov, and others [2].

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