Wedderburn-type theorems for operator algebras: traditional and "quantized" homological approaches

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ABSTRACT

Let H be a Hilbert space, A be a subalgebra of $\mathcal{B}(H)$. We call A a Wedderburn algebra if there exists a "canonical" decomposition

$$H = \oplus\{H'_\nu \otimes H''_\nu; \quad \nu \in \Lambda\}$$

such that A consists of all $a \in \mathcal{B}(H)$ such that, for any $\nu \in \Lambda$, the restriction of A onto $H'_\nu \otimes H''_\nu$ has the form $b_\nu \otimes 1$ for some $b_\nu \in H'_\nu$.

Which conditions of the homological nature can distinguish Wedderburn algebras? We suggest to consider the so-called *spatial projectivity* of A, that is the projectivity of the A-module H. There are two versions of this property: the traditional one, based on the usual notion of a Banach module, and the "quantum" one, based on that of the quantum, or operator, module.

We show that *a given von Neumann algebra is Wedderburn if and only if it is quantum spatially projective* whereas *traditionally projective von Neumann algebras are exactly Wedderburn algebras with the following additional property: for any ν in the canonical decomposition of H we have* $\min\{\dim H'_\nu, \dim H''_\nu\} < \infty$.

Departing from this two-fold assertion, we describe at first all spatially projective (in both senses) operator C^*-algebras, and then all projective Hilbert modules over (arbitrary) C^*-algebras. Some parts of these results can be extended to certain classes of non-selfadjoint operator algebras.

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