

Subordination-Preserving Integral Operators for Analytic Functions

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ABSTRACT

Let $H(U)$ be the space of analytical functions in the unit disk U . In the first part, considering the integral operator $A_h : \mathcal{K} \rightarrow H(U)$, $\mathcal{K} \subset H(U)$, defined by $A_h(f) = F$, where

$$F(z) = \left[\beta \int_0^z f^\beta(t) h^{-1}(t) h'(t) dt \right]^{1/\beta},$$

we determine conditions on h and g such that

$$\left(\frac{zh'(z)}{h(z)} \right)^{1/\beta} f(z) \prec \left(\frac{zh'(z)}{h(z)} \right)^{1/\beta} g(z) \text{ implies } A_h(f) \prec A_h(g).$$

We obtain, in particular, an extension of a Goluzin result and study some special cases of the main theorem obtained for different choices of h .

In the second part, for the operator $A_{\beta,\gamma} : \mathcal{K} \rightarrow H(U)$, $\mathcal{K} \subset H(U)$, defined by

$$A_{\beta,\gamma}(f)(z) = \left[\frac{\beta + \gamma}{z^\gamma} \int_0^z f^\beta(t) t^{\gamma-1} dt \right]^{1/\beta}$$

and $\beta, \gamma \in \mathbb{C}$, we determined conditions on $g(z)$, β and γ such that

$$z \left[\frac{f(z)}{z} \right]^\beta \prec z \left[\frac{g(z)}{z} \right]^\beta \text{ implies } z \left[\frac{A_{\beta,\gamma}(f)(z)}{z} \right]^\beta \prec z \left[\frac{A_{\beta,\gamma}(g)(z)}{z} \right]^\beta$$

and we presented some particular cases of our main result.

References

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