Section 11: Complex Analysis | Poster number 221

New classes of definite integrals

Maximilian Antimirov*, Riga Technical University.
Juri Antimirov, Software Developer, MGI Software.

ABSTRACT

We consider an integral of the form

$$I_1 = \int\limits_{-\infty}^{+\infty} \frac{P_n(e^x)}{Q_m(e^x)} \frac{dx}{(\sinh x + ax)^2 + a^2\pi^2}, \tag{1}$$

where $a = \text{const},\ a \neq 0$; $P_n(x)$ and $Q_m(x)$ are real polynomials of the real variable x of degree n and m, respectively; $m \geq n-1$; $Q_m(e^x) \neq 0$ for real x. The integral (1) is evaluated in closed form by integrating of some function of complex variable in complex plane along the closed contur and using the residue theorem. For using the residue theorem it is proved that at $a > 0$ and $a \neq 1$ in the trip $0 \leq \text{Im} z \leq 2\pi$ there are only 3 simple zeros of the function $\varphi(z) = \sinh z + a(z - \pi i)$: $z_1 = \pi i, z_{2,3} = i(\pi \pm \beta)$ if $a \in (0,1)$, and $z_{2,3} = i\pi \pm \gamma$, if $a > 1$. Here β is the positive root of equation $\sin\beta = a\beta$, $0 < \beta < \pi$ and γ is the positive root of equation $\sinh\gamma = a\gamma$ (location of the roots $\sin\beta = a\beta$ see also in [1]).

References

[1] M.Ya. Antimirov, A.A. Kolyshkin, R. Vaillancourt, *Complex Variables*, AP, San Diego, London, Boston, New York, Sydney, Tokyo, Toronto, 1998.

Keywords: *residue theorem, roots of transcendental equation*

Mathematics Subject Classification: *30*

Contact Address: antimir@egle.cs.rtu.lv