Weakly dense ideals in some F-algebras of holomorphic functions

Romeo Meštrović, University of Montenegro, Maritime faculty, Kotor, Montenegro, Yugoslavia.

ABSTRACT

For $p > 1$, the class N^p, introduced by I.I. Privalov, consists of those holomorphic functions f on the open unit disk D in the complex plane for which $(\log^+ |f(z)|)^p$ has a harmonic majorant on D. M. Stoll showed that the space N^p with the topology given by the metric ρ_p defined by

$$\rho_p(f, g) = \Big(\int_0^{2\pi} \big(\log(1 + |f^*(e^{i\theta}) - g^*(e^{i\theta})|) \big)^p \, \frac{d\theta}{2\pi} \Big)^{1/p}, \quad f, g \in N^p,$$

becomes an F-algebra.

We consider the closed ideals in the algebras N^p. By a result of N. Mochizuki, the closed ideals in N^p are the principal ideals generated by inner functions. We prove that if I is a closed weakly dense ideal in N^p, then I is generated by a singular inner function. Conversely, if S is a singular inner function whose associated singular measure has modulus of continuity $o\left(t^{\frac{p-1}{p}}\right)$, then the ideal SN^p is weakly dense in N^p. As an application, we show that for such a singular inner function S, the quotient space N^p/S is an F-space with trivial dual, and hence N^p is not locally convex.

Keywords: *F-algebra N^p, closed ideal, weakly dense ideal, singular inner function*

Mathematics Subject Classification: *30H05*

Contact Address: `fzpkotor@cg.yu`