Uniform approximation by polyanalytic polynomials and related problems

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ABSTRACT_

Let n be a natural number. A function f is called n-analytic (polyanalytic of order n) in an open set $G \subset \mathbb{C}$ if $\overline{\partial}^n f = 0$ in G, where $\overline{\partial}^n$ is the n-power of the Cauchy-Riemann operator. We consider the following problem: under what conditions on a compact set $X \subset \mathbb{C}$ each function which is continuous on X and n-analytic in the interior of X can be uniformly approximated on X by n-analytic polynomials. This approximation problem is closely related to the Dirichlet problem for polyanalytic functions.

In the present poster some necessary and sufficient conditions for uniform approximability of functions by polyanalytic polynomials of order $n \ge 2$ on plane compact sets of special type are considered (see [1,2,3]). These conditions are formulated in terms of special analytic characteristic of the compact set under consideration. It means that the solution of the approximation problem mentioned above would apparently depend on some very complicated analytic properties of X. These properties in their turn would depend on n. A correlation between the problem in question and the Dirichlet problem for bianalytic (i.e. 2-analytic) functions is considered as well.

References

[1] K.Yu. Fedorovski. Uniform *n*-analytic polynomial approximations of functions on rectifiable contours in \mathbb{C} . Math. Notes. 1996. v.59. No.4, pp.435–439.

[2] K.Yu. Fedorovski. On uniform approximation by polyanalytic polynomials on compact subsets of the plane. Annal. Univ. M.Curie-Sklodowska. 1999. v.53, No.3, pp.27–39.

[3] J.J. Carmona, K.Yu. Fedorovski and P.V. Paramonov. On uniform approximation by polyanalytic functions and the Dirichlet problem for bianalytic functions. Centre de Recerca Matematica. Preprint No. 415. Juliol, 1999. 18p.

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