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Best constants in the Burkholder-Rosenthal-type inequalties for multilinear forms

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ABSTRACT

We prove the explicit expressions for the best constants in the following analogues of the Burkholder-Rosenthal exact moment inequalities for multilinear forms obtained in a general form in [1]:

$$E(\sum_{1\le i_1<...<i_m\le n} X_{i_1}...X_{i_m})^t \le C_1(t,m,n)max(C_n^m(EX_1^t)^m, (C_n^m)^t(EX_1)^{mt})$$

for independent identically distributed nonnegative random variables $X_1, ..., X_n$ with finite tth moment, $t \ge 1$;

$$E\mid \sum_{1\le i_1<...<i_m\le n} X_{i_1}...X_{i_m}\mid^t \le C_2(t,m,n)max(C_n^m(E\mid X_1\mid^t)^m, (C_n^m)^{t/2}(EX_1^2)^{mt/2})$$

for independent symmetric identically distributed random variables $X_1, ..., X_n$ with finite tth moment, $t \ge 2$. In the case of linear forms we show that if $1 \le l \le m$, $k_1 = 1 < k_2 < ... < k_l$ are arbitrary elements of the set $\{2s-1, s = 1, 2, ..., m\}$ and $C^*_{k_1,...,\ k_l}$ denote the best constants in the Burkholder-Rosenthal inequality

$$E\mid \sum_{i=1}^n X_i\mid^{2m} \le C_{k_1,...,\ k_l} max(\sum_{i=1}^n E\mid X_i\mid^{2m}, (\sum_{i=1}^n EX_i^2)^m)$$

for independent random variables $X_1, ..., X_n$ with moments of orders $k_1, k_2, ..., k_l$ equal zero, then $C^*_{k_1,...,\ k_l} \le T_{k_1,...,\ k_l}$ where $T_{k_1,...,\ k_l}$ are the numbers of partitions of a set consisting of $2m$ elements into parts the number of elements in which is not equal to $k_1, ...,\ k_l$. The latter bounds are exact in the extremal cases of random variables with zero mean, random variables with m zero first odd moments and nonnegative random variables (e.g., [1], [2]).

References

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