

Least squares method estimation in bisexual Galton-Watson branching processes with immigration

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ABSTRACT

Introduced by Daley(1968), the Bisexual Galton-Watson Process (BGWP) is a two type branching model $\{(F_n, M_n), n = 1, 2, \dots\}$ defined:

$$Z_0 = N \geq 1, (F_{n+1}, M_{n+1}) = \sum_{i=1}^{Z_n} (f_{ni}, m_{ni}), Z_{n+1} = L(F_{n+1}, M_{n+1}), n = 0, 1, \dots$$

with the empty sum defined to be $(0, 0)$. Where f_{ni} (m_{ni}) represents the number of females (males) produced by the i th mating unit in the n th generation, being $\{(f_{ni}, m_{ni}), i = 1, 2, \dots; n = 0, 1, \dots\}$ a sequence of i.i.d., non-negative, integer-valued random variables and the mating function $L: \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is assumed to be monotonic non decreasing in each argument, integer-valued for integer-valued arguments and such that $L(x, y) \leq xy$. Thus F_n (M_n) will be the number of females (males) in the n th generation, which form $Z_n = L(F_n, M_n)$ mating units. These mating units reproduce independently through the same offspring distribution for each generation. In the last years, this branching model has received considerable attention in the scientific literature, however, unlike asexual Galton-Watson process, modified bisexual Galton-Watson models have not been, until now, sufficiently developed. With respect to this, recently, bisexual Galton-Watson models allowing immigration of females and males or mating units, have been introduced by González, Molina and Mota(2000), where some probabilistic properties have been investigated.

This paper is devoted to look at the estimation of the offspring and immigration mean vectors and the growth rate for the new introduced models. Estimators, based on the least squares method, are proposed and their asymptotic properties are investigated. We also prove the nonexistence of consistent estimators for the immigration mean vector. As illustration, some simulated examples are given.

References

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