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On separable families of probabilities and sufficiency

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ABSTRACT

The aim of this work is to provide an interesting property about separable (in topological sense) families of probabilities. We start with an statistical experiment $(\Omega, \mathcal{A}, \mathcal{P})$, where Ω is a set, $\mathcal{A}$ a σ-field of $\mathcal{P}(\Omega)$ and $\mathcal{P}$ a family of probabilities on $(\Omega, \mathcal{A})$. Suppose that the topological space $(\mathcal{P}, \mathrm{d})$ is separable, where d is the metric defined in the following way:

$$\mathrm{d}(P, Q) = \sup_{A \in \mathcal{A}} |P(A) - Q(A)|, \quad P, Q \in \mathcal{P}.$$

Then, on the one hand, it can be shown (Strasser 1985) that there exists a probability P_o that dominates the family $\mathcal{P}$ and a σ-field $\mathcal{B} \subset \mathcal{A}$ so that, for every $P \in \mathcal{P}$, there exists a $\mathcal{B}$-measurable version of dP/dP_o.

On the other hand, it is well known (Barra 1971) that, for every dominated statistical experiment, there exists only one (essentially) minimal sufficient σ-field. Besides, it can be described in an explicit way. If both statements are related, we shall conclude that, if $(\mathcal{P}, \mathrm{d})$ is separable, then there exists only one (essentially) minimal sufficient σ-field; moreover, it is separable (in the sense that it is generated by a countable collection of events in $\mathcal{A}$). We shall illustrate this question with an example.

References

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