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On test and generalized functions spaces and stochastic integral in γ–analysis

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ABSTRACT

Let $\mathcal{D}$ be the classical (real) Schwartz space, $\mathcal{D}'$ be its dual one constructed by the chain $\mathcal{D}' \supset L^2(\mathbb{R},\sigma) \supset \mathcal{D}$, where σ is a non–atomic positive Radon measure. The γ–measure on $\mathcal{D}'$ can be characterized by its Laplace transform

$$l_{\mu^\sigma}(\theta) = \int_{\mathcal{D}'} e^{\langle x,\theta\rangle}\mu^\sigma(dx) = \exp\{-\langle 1, \log(1-\theta)\rangle\},\ 1 > \theta \in \mathcal{D},$$

where $\langle\cdot,\cdot\rangle$ denotes the dual pairing generated by the scalar product in $L^2(\mathbb{R},\sigma)$. By analogy with Gaussian and Poisson analysis, one can introduce the Wick exponential

$$:\exp(x;\theta):\stackrel{def}{=} \frac{e^{\langle x,\alpha(\theta)\rangle}}{l_{\mu^\sigma}(\alpha(\theta))} = \sum_{n=0}^{\infty}\frac{1}{n!}\langle P_n^{\mu^\sigma}(x), \theta^{\otimes n}\rangle,\ P_n^{\mu^\sigma}(x) \in \mathcal{D}_{\mathbb{C}}^{'\widehat{\otimes}n},$$

where $\alpha(\theta) = \theta/(\theta-1)$, subindex '$\mathbb{C}$' denotes the complexification of space, $\widehat{\otimes}$ denotes the symmetric tensor product. As it was proved in [1], the polynomials $\langle P_n^{\mu^\sigma}(x), \varphi^{(n)}\rangle$, $\varphi^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes}n}$ are orthogonal in the sense that

$$\int_{\mathcal{D}'}\langle P_n^{\mu^\sigma}(x), \varphi^{(n)}\rangle\langle P_m^{\mu^\sigma}(x), \psi^{(m)}\rangle\mu^\sigma(dx) = \delta_{mn}n!\langle\varphi^{(n)},\psi^{(n)}\rangle_{Ext},$$

where $\langle\cdot,\cdot\rangle_{Ext}$ is the special pairing described e.g. in [1].

In this communication we construct an analog of Gaussian and Poisson infinite–dimensional analysis for γ–measure (now the polynomials $\langle P_n^{\mu^\sigma}(x), \varphi^{(n)}\rangle$ play the role of Hermite polynomials in Gaussian analysis or Charlier polynomials in Poisson analysis). More exactly, we construct and study test and generalized functions spaces, integral transforms, the Wick calculus, an analog of stochastic integral and stochastic equations with Wick type nonlinearity. Note that since $\langle\cdot,\cdot\rangle \neq \langle\cdot,\cdot\rangle_{Ext}$, the '$\gamma$–analysis' can not be constructed by the 'classical' way.

Reference

[1]. Kondratiev, Yu. G., J. Luis da Silva, Streit, L., Us, G. F. *Analysis on Poisson and Gamma spaces*, Quantum Probab. and Related Topics, v. 1, pp. 91–117, 1998.

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