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On convergence to statistic equilibrium in two-temperature problem for wave equation with mixing

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ABSTRACT

We consider wave equations in the whole space R^3. We assume that the initial datum $Y_0 = (Y_0^0(x), Y_0^1(x))$ is a random function in an appropriate functional Fréchet space $\mathcal{H}$, Y_0 has zero expectation and finite mean density of the energy. Moreover, we assume a mixing condition of Ibragimov-Linnik type. Roughly speaking, the random values $Y_0(x)$ and $Y_0(y)$ are asymptotically independent as $|x-y| \to \infty$. We assume that the initial correlation functions $Q_0^{ij}(x,y) \equiv EY_0^i(x)Y_0^j(y)$, $i,j = 0,1$, and some of its derivatives are continuous and have a decay as $|x-y| \to \infty$. At last, we assume that correlation functions $Q_0^{ij}(x,y) = q_-^{ij}(x-y)$, if $x_3, y_3 <-a$ and $Q_0^{ij}(x,y) = q_+^{ij}(x-y)$ if $x_3, y_3 > a$. Here a is a positive constant, and $q_\pm^{ij}(x-y)$ are the correlation functions of some Gaussian translation-invariant measures $\mu_\pm$ on $\mathcal{H}$.

We study the distribution μ_t of the random solution at the moments $t \in R$. Our main result means a convergence to a statistic equilibrium, i.e. a weak convergence of the measures $\mu_t \rightharpoonup \mu_\infty$, $t \to \infty$, where μ_∞ is a Gaussian space-homogeneous measure. We obtain the explicit formulas for the correlation functions of the limit measure μ_∞. We extend the results to the equations with variable coefficients, which are constant outside a finite region. We apply our results to the case of Gibbs measures $\mu_\pm = g_\pm$ corresponding to two different temperatures $T_\pm$. We show that the limit energy current *formally* is $+\infty \cdot (0,0,T_+ - T_-)$ for the Gibbs measures, and it is finite $\sim (0,0,T_+ - T_-)$ for the convolution with a test function.

Keywords: *wave equation, random solution, statistic equilibrium*

Mathematics Subject Classification: *60F17, 82B09, 82C*

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