Calculus of zonal polynomials and some applications in Multivariate Analysis

José Rodríguez Avi, Departamento de Estadística e I.O. Universidad de Jaén.
Antonio José Sáez Castillo*, Departamento de Estadística e I.O. Universidad de Jaén.
Antonio Conde Sánchez, Departamento de Estadística e I.O. Universidad de Jaén.

ABSTRACT

Zonal polynomials have been used extensively in the field of Mathematical Statistics associated with the expressions of densities and distributions of quadratics forms in normal samples.

These polynomials are functions with argument a matrix and their values are real numbers. They extend many of the properties that powers of one real variable verify to the multivariate case.

Nevertheless, general formulas for zonal polynomials are unknown. They are defined as eigenfunctions of a differential operator (the Laplace-Beltrami operator) and because of it, they only can be calculated in a iterative way through algorithms.

Results obtained after the implementation of one of these algorithms are shown. The algorithm implemented is be able to calculate zonal polynomials of high degrees; it is due to James, although a change has been introduced for decreasing the time spent in computations.

Some applications are shown. For instance, distributions of highest and lowest latent roots of a Wishart; multivariate and univariate distributions generated by hypergeometric functions of matricial argument (these functions are series of zonal polynomials which extend the family of generalized hypergeometric functions).

References

[1] Muirhead, R. J. (1982). Aspects of multivariate statistical theory. *Wiley.*

[2] Rodríguez, J., Gutiérrez, R. and Sáez, A.J. (1999). "Construction of multivariate discrete distributions trough the $_2F_1$ function of matricial argument". In *Applied Stochastics Models and Data Analysis, Proc. IX International Symp. Lisboa* (ed. H. Barcelar-Nicolau, F. Costa-Nicolau and J. Jansen) pp. 194–199. I.N.E. Portugal.

[3] Gutiérrez R., Rodríguez J., Sáez, A. J. Approximation of hypergeometric functions of matricial argument through their development in series of zonal polynomials. *Submitted*