Limit theorems for random combinatorial structures

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ABSTRACT

We deal with probabilistic problems of random combinatorial structures such as assemblies and multisets (see [1] for definitions). They include permutations, mappings of a finite set into itself, polynomials over finite field, mapping patterns, elements of arithmetical semigroups *et cetera*. If a random structure σ of size n, taken with some probability $\nu_n(\{\sigma\})$, has $k_j = k_j(\sigma)$ components of size j, $1 \leq j \leq n$, then the relation $1k_1 + \cdots + nk_n = n$ involves a rather specific dependence of r.vs k_j. This leads to many phenomenon in limiting behaviour of distributions of the statistics of the form $H_m(\sigma) := h_1(k_1) + \cdots + h_m(k_m)$, where $h_j(k) \in \mathbf{R}$, $h_j(0) = 0$, $m \leq n$, and $n \to \infty$. The cases when $m = n$ and $m = m_n(t)$, where $t \in [0, 1]$ is a time parameter, comprise the main target of our interest. The second problem is treated in the frames of general functional limit theorems in the Skorokhod space $\mathbf{D}[0, 1]$. The statistics $H_m(\sigma)$ may also be considered as given on the set $\mathfrak{P}$ of partitions of the natural number n. Then the frequences ν_n induce probability measures on $\mathfrak{P}$. The Ewens sampling formula, induced by some weighted measure ν_n on the symmetric group, is a typical example.

We examine the problems by probabilistic and analytic methods. The first one exploits the representation of the joint distribution of $\bar{k}$ by that of a vector with independent coordinates subject to some linear condition. The fundamental estimates of the total variation distance between combinatorial and independent processes (see [1] and more recent papers by R.Arratia, D.Stark, and S.Tavaré) allow to consider the statistics above up to $m = o(n)$. In the poster we will demonstrate the influence of the strongly dependent summands $h_j(k_j)$ over interval $[\varepsilon n, n]$, $0 < \varepsilon < 1$. This has been afforded using an analytic method based upon generating series of combinatorial structures.

One dimensional limit theorems for $H_n(\sigma)$ as well as functional limit theorems for $H_{m_n(t)}(\sigma)$ will be presented in the poster. Shortly speaking, for many combinatorial structures, we have necessary and sufficient conditions when the combinatorial processes weakly converge to stochastic processes with independent increments and we do have instances when the limit processes are not even infinitely divisible.

Reference

1. R.Arratia and S.Tavaré, Independent process approximations for random combinatorial structures, *Advances in Math.*, 1994, 104, 1, 90–154.

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