

Exchangeable Random Orders and Almost Uniform Distributions

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ABSTRACT

We consider exchangeable random total orders on \mathbf{N} and show that they form a simplex whose set of extreme points, the so-called totally ordered paintbox processes, is homeomorphically parametrized by the “almost uniform” distributions on the unit interval, i.e. by the probability measures w on $[0,1]$ whose distribution functions are w -almost surely the identity. This is work done by myself and Paul Ressel, entitled “Exchangeable random orders and almost uniform distributions”, to appear in the Journal of Theoretical Probability. It is stimulated by Alexander V. Gnedin, “The representation of composition structures”, Annals of Probability 25, No. 3, 1437-1450 (1997), and it is a follow-up paper of “Random partitions by semigroup methods” in the Semigroup Forum Vol. 59 (1999) 126-140, also by myself and Paul Ressel. The underlying background goes back to Peter Donnelly and Paul Joyce (“Consistent ordered sampling distributions: characterization and convergence”, Adv. Appl. Prob. 23, 229-258 (1991)) and to John Frank Charles Kingman (“The coalescent”, Stochastic Processes and their Applications 13 (1982) 235-248 and “The representation of partition structures”, Journal of the London Mathematical Society 18 (1978) 374-380). Our overall approach is analogous to that used already in our first paper to derive Kingman’s results on exchangeable random partitions. That is, we do not consider finite dimensional approximations, avoiding thus combinatorial arguments as in Donnelly/Joyce, Gnedin and Kingman. Instead we use some harmonic analysis on semigroups, in particular a suitably constructed “positivity forcing” map. For the proof of our main result (Theorem 4) we use the method of moments, applied to a sequence of random measures (empirical distributions) on $[0,1]$ (a bootstrap method). Only the almost sure convergence of these random measures (Theorem 5) requires the martingale convergence theorem.

Keywords: *exchangeability, exchangeable random structures, almost uniform distributions, simplex, Bauer simplex, extreme points, paintbox, harmonic analysis, semigroups, positive definite functions, positivity forcing, empirical distributions, bootstrap.*

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