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Boundedness and compactness weighted criteria for the operators with positive kernels

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ABSTRACT

Necessary and sufficient conditions for weight v defined on $R_a^+ \equiv (0,a)\times(0,\infty)$, $0<a\le\infty$, guaranteeing the boundedness (compactness) of the operator

$$Kf(x,t)=\int_0^x k(x,y,t)f(y)dy,\ \ (x,t)\in R_a^+,\ k\ge 0,$$

from $L^p(0,a)$ into $L_v^q(R_a^+)$ are found, where $0<p,q<\infty$ and $p>1$. The non-compactness measure of K is also estimated.

Analogous problems for the generalized Riemann-Liouville operator

$$T_\alpha f(x,t)=\int_0^x f(y)(x-y+t)^{\alpha-1}dy,\ \ x\in(0,\infty),\alpha>1/p,$$

were solved in [4]. For the classical Riemann-Liouville operator $R_\alpha f(x)\equiv T_\alpha f(x,0)$ see [3].

The boundedness weighted criteria from L_w^p to L_v^q, $1<p<q<\infty$, for the oparator with positive kernel were derived in [1], Chapter 3 (For $0<q\le p<\infty$, $p>1$ and $w\equiv 1$ see [2]).

References

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