3rd European Congress of Mathematics
Poster sessions

Section 13: Real Analysis — Poster number 144

Corner point Gibbs phenomena for Fourier series in two dimensions

Gilbert Helmberg, University of Innsbruck, Institut für Mathematik.

ABSTRACT

Let V be a convex neighbourhood of the origin contained in the square $Q = \{(x, y) \in \mathbb{R}^2 : |x| \le \pi, |y| \le \pi\}$. Let A be a 'sector' of V bounded by two convex or concave curves intersecting at the origin. Let f be a function on Q, smooth on A and on $V \setminus A$ but having a jump discontinuity at the origin, whose coordinate sections (on $x = const$, resp. $y = const$) have uniformly bounded variation. Under essentially these conditions the partial sums $S_{n,n}$ of the Fourier series of f display for $n \to \infty$ at the origin a corner point Gibbs phenomenon only depending on the slopes of the boundary curves of A at the origin and on the jump levels of $f(x, y)$ as (x, y) approaches $(0, 0)$ within A resp. $V \setminus A$. This Gibbs phenomenon manifests itself in the convergence of $S_{n,n}(\frac{x}{n}, \frac{y}{n}; f)$ as $n \to \infty$ for all $(x, y) \in \mathbb{R}^2$ to a non-constant function $\tilde{S}$ with an overshoot of up to 37,4% of half the jump size.

If the tangents of the above mentioned boundary curves at the origin have slopes respectively 0 and c (pointing into the upper half plane) and if the function f jumps at the origin from level 0 to level 1, then putting $x_1 = x - \frac{y}{c}$ and $\bar{c} = \min(|c|, 1)$ the limit function $\tilde{S}$ is given by

$$\tilde{S}(x, y, c) = \lim_{n\to\infty} S_{n,n}(\frac{x}{n}, \frac{y}{n}; f) = \frac{1}{4} + \frac{1}{2\pi}\int_0^y \frac{\sin s}{s} ds + \frac{1}{2\pi}\int_0^{x_1\bar{c}} \frac{\sin t}{t} dt + \frac{1}{2\pi^2}\int_0^{x_1} \frac{\sin t}{t} dt \int_{\frac{ty}{cx_1}-y}^{\frac{ty}{cx_1}+y} \frac{\sin s}{s} ds -$$

$$- \frac{1}{2\pi^2}\left\{\int_0^{x_1\bar{c}} \frac{\cos t}{t} dt \int_{y-\frac{ty}{cx_1}}^{y+\frac{ty}{cx_1}} \frac{\cos s}{s} ds + \int_{x_1\bar{c}}^{x_1} \frac{\cos t}{t} dt \int_{\frac{ty}{cx_1}-y}^{\frac{ty}{cx_1}+y} \frac{\cos s}{s} ds\right\}.$$

In particular,

$$\tilde{S}(0, 0, c) \;=\; \lim_{n\to\infty} S_{n,n}(0, 0; f) = \frac{1}{4} - \frac{1}{2\pi^2}\left\{\int_0^{\bar{c}} \frac{1}{t}\log\left(\frac{c+t}{c-t}\right) dt + \int_{\bar{c}}^{1} \frac{1}{t}\log\left(\frac{t+c}{t-c}\right) dt\right\}.$$

A related phenomenon concerning Laplace series on the sphere has been studied by H. Weyl (Rend. Circ. Mat. Palermo **29, 30** (1910)).

References

G. Helmberg, A corner point Gibbs phenomenon for Fourier series in two dimensions, J. Approximation Theory **100** (1999), pp. 1–43.

G. Helmberg, Localisation of a corner point Gibbs phenomenon for Fourier series in two dimensions. Submitted for publication.

Keywords: *Gibbs phenomenon, two-dimensional Fourier series, jump discontinuity, local properties in two dimensions, Dirichlet kernel.*

Mathematics Subject Classification: *42B99*

Contact Address: gilbert.helmberg@uibk.ac.at