3rd European Congress of Mathematics
Poster sessions

Section 13: Real Analysis — Poster number 288

On the relative fundamental solutions for a second order differential operator on the Heisenberg group

Tomás Godoy, Facultad de Matemática, Astronomía y Física, Universidad Nacional de Córdoba (Argentina).
Linda Saal*, Facultad de Matemática, Astronomía y Física, Universidad Nacional de Córdoba (Argentina).

ABSTRACT

Let H_n be the $2n+1$ dimensional Heisenberg group, viewed as $C^n \times R$, let p, q be two non negative integers satisfying $p+q=n$. Let $L=\sum_{j=1}^{p}\left(X_j^2+Y_j^2\right)-\sum_{j=p+1}^{n}\left(X_j^2+Y_j^2\right)$, where $\{X_1, Y_1, ...X_n, Y_n, T\}$ denotes a suitable standard basis of the Lie algebra of H_n adapted to the action of $U(p,q)$ on H_n given by $g(z,t)=(gz,t)$, $g \in U(p,q)$, $(z,t) \in C^n \times R$. A tempered distribution $\Phi \in S'(H_n)$ is called a relative fundamental solution for L if $L(f * \Phi) = L(f) * \Phi = f - \pi(f)$ for all $f \in S(H_n)$ where π is the orthogonal projection on $K = Ker(L)$. In [M-R1] and in [M-R2] it is proved that there exist relative fundamental solutions for a wide class of second order differential operators that includes L. There, it is given also an explicit formula for L in the case $p=q=1$. In this work we compute explicitly a such Φ for arbitrary p, q. The main tool used are a spectral decomposition on $L^2(H_n)$ associated to L and iT obtained in [G-S], and the description of $S'\left(R^{2n}\right)^{O(p,q)}$ given in [T], adapted to describe $S'\left(C^n\right)^{U(p,q)}$. In [T], is introduced the space h of the functions $\varphi(\tau)=\varphi_1(\tau)+\tau^{n-1}H(\tau)\varphi_2(\tau)$ with $\varphi_1, \varphi_2 \in S(R)$, where H is the Heaviside function. h, provided with a suitable topology is a Frechet space. A very specific operator $N: S(C^n) \to h$ is introduced there, whose adjoint $N': h' \to S'\left(C^n\right)^{U(p,q)}$ is an isomorphism.

For $z=(z_1,...,z_n) \in C^n$, let $B(z)=\sum_{j=1}^{p} z_j^2 - \sum_{j=p+1}^{n} z_j^2$. For $f \in S(H_n)$ and $\tau, t \in R$, let $Nf(\tau,t) = N(f(.,t))(\tau)$. In this setting, we prove that a relative fundamental solution Φ for L is given by

$$\langle \Phi, f\rangle = c\int_{R^2} \frac{sg(\tau)}{(\tau^2+16t^2)^{n/2}}\left(Nf(\tau,t)-\sum_{j=0}^{n-2}\frac{\partial^j(Nf)}{\partial\tau^j}(0,t)\,\tau^j\right)d\tau dt+\sum_{j=0}^{n-2} c_j\langle T_j, f\rangle$$

Here, c is the constant obtained by Folland for the case $p=n$, $q=0$. The distributions T_j, supported on $\{(z,t) \in C^n \times R : B(z)=0,\ t=0\}$, and the constants c_j are explicitly computed.

References

[G-S] Godoy, T., Saal, L.: L^2 spectral decomposition on the Heisenberg group associated to the action of $U(p,q)$. Pacific J. of Math., Vol 193, No. 2, 327-353 (2000).

[M-R1] Muller, D., Ricci, F. Analysis of second order differential operators on Heisenberg groups I, Invent. Math. 101, 545-582 (1990).

[M-R2] Muller, D., Ricci, F. Analysis of second order differential operators on Heisenberg groups II, J. Funct. Anal. Vol 108, No 2, 296-346, September 1992.

[T] Tengstrand, A. Distributions invariant under an orthogonal group of arbitrary signature, Math. Scand., 8, 201-218 (1960).

Keywords: *Heisenberg group, Relative fundamental solutions, Heisenberg Laplacian*

Mathematics Subject Classification: *43A80*

Contact Address: `godoy@mate.uncor.edu`