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On rearrangements of orthogonal systems

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ABSTRACT

We consider orthonormal systems (ONS) $\{f_n\}_0^\infty$ of functions $f_n \in L^2[0,1]$. Such a system is called a system of convergence if any orthogonal series (OS)

$$\sum_{n=0}^{\infty} c_n f_n(x), \quad \{c_n\} \in l^2, \tag{1}$$

converges almost everywhere on $[0,1]$. The following classical problem is known for a long time and is called the Kolmogorov–Men'shov problem (see, e.g., [2], p. 435, and references therein): does there exist a rearrangement of any ONS that leads to a system of convergence? The problem is not solved up to now.

Many authors, following D.E. Men'shov [3], investigated modified problems replacing the condition of convergence by that of summability by various summability methods.

We consider the modified problem using summability methods from the class $\Phi\Lambda$ that contain many well-known particular summability methods (see [4]). A method (φ, λ) of the class $\Phi\Lambda$ is defined by a function $\varphi = \varphi(t)$ $(t \geq 0)$ with $\varphi(0) = 1$ and $\varphi(\infty) = 0$ and a sequence $\lambda = \{\lambda_n\}$, $0 \leq \lambda_n \uparrow \infty$. The OS (1) is summable by the method (φ, λ) to $f(x)$ at a point $x \in [0,1]$ if

$$\lim_{t \to 0} \sum_{n=0}^{\infty} c_n f_n(x) \varphi(\lambda_n t) = f(x).$$

Theorem. *Let φ be a convex function on $[h, \infty)$ with $\varphi' \in L^p[h+1, \infty)$, where $p > 1$. Then for any ONS $\{f_n\}$ there is a rearrangement $\{f_{n_k}\}$ such that any OS $\sum c_k f_{n_k}(x)$ is (φ, λ)-summable a.e. on $[0,1]$.*

References.

[1] D.E. Men'shov, Selected Works, "Faktorial", Moscow, 1997 (Russian).

[2] P.L. Ul'yanov, Development of D.E. Men'shov's results in the theory of orthogonal series, Appendix in [1], 425–451.

[3] D.E. Menchoff, Bull. Soc. Math. France 64 (1936), 147–170.

[4] O.A. Ziza, Summability of Orthogonal Series, URSS, Moscow, 1999.

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