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On the approximation of the functions by algebraic polynomials

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ABSTRACT

For any $r > 0$ and $p = 1$ or $p = \infty$, we consider the class W_p^r of functions $f(x)$ represented in the form $f(x) = \frac{1}{\Gamma(r)} \int_{-1}^{x} (x-t)^{r-1} \phi(t)\, dt + P_m(x)$, where $\Gamma(r)$ is the Euler gamma function, $\|\phi\|_1 \le 1$, if $p = 1$, and $|\phi(t)| \le 1$ almost everywhere if $p = \infty, P_m(x)$ is an algebraic polynomial of degree $m \le [r] - 1$.

Set

$$S_\rho(f)(x) = \frac{1}{\pi} \int_{-1}^{1} \frac{f(t)}{t-x} \rho(t)\, dt,$$

where integrals is considered in the sense of the principal value and $\rho(t)$ is the nonnegative integrable functions. The following statements hold.

Theorem 1. For any $r > 0$ and any function $f \in W_\infty^r$ there exists sequence of the algebraic polynomials $\{P_n(x)\},\ \ n = [r]+1, [r]+2, ...,$ such that

$$|S_\rho(f)(x) - P_n(x)| \le \frac{\tilde{K}_r}{n^r} (\sqrt{1-x^2})^{r+1} + O\left(\frac{\ln n}{n^{r+1}} (\sqrt{1-x^2} + 1/n)^r \right),$$

where $\rho(t) = \sqrt{1-t^2}$ and $\tilde{K}_r$ is the best approximation by a constant in L_1 to the functions

$$\widetilde{D}_r(t) = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\sin(kt - \frac{r\pi}{2})}{k^r}.$$

Theorem 2. For any $r \in N$ and any $f \in W_1^r$ there exists the algebraic polynomial $P_n(x)$ of degree $n \ge r-2$ such that

$$\|S_\rho(f) - P_n(x)\|_1 < \frac{\tilde{K}_r (n-r+1)!}{(n+1)!},$$

where $\rho(t) = 1/\sqrt{1-t^2}$.

The asymptotically exact inequalities for the pointwise and L_1-approximation of the functions $S_\rho(f)$ by polynomials are obtained and for other cases.

Let $W^r H^\omega,\ \ r = 1, 2, ...,$ be a class of functions on [-1,1] for which the r-th derivative satisfys the condition: $|f^{(r)}(x_1) - f^{(r)}(x_2)| \le \omega(|x_1 - x_2|), x_1, x_2 \in [-1,1]$, where $\omega(t)$ is given modulus of continuity.

Theorem 3. Let $\omega(t)$ be a convex modulus of continuity such that $t\omega'(t)$ is nondecreasing function. Then, for any $f \in W^r H^\omega$ there exists sequence of the algebraic polynomials $\{Q_n^r(f;x)\},\ \ n = r, r+1, ...,$ such that

$$|f(x) - Q_n^r(f;x)| \le \frac{K_r}{2} (\frac{\sqrt{1-x^2}}{n})^r \omega\left(\frac{2K_{r+1}}{K_r n} \sqrt{1-x^2} \right) + C_r \frac{\Delta_n^{r-1}(x) \omega(\Delta_n(x)) \ln n}{n^{r+1}},$$

where K_r are the Favard constants and $\Delta_n(x) = \frac{\sqrt{1-x^2}}{n} + \frac{1}{n^2}$.

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