Analytical solution of one magnetohydrodynamic problem for cylindrical region

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ABSTRACT

The round channel is located in the region $0 \le \tilde{r} \le R\,,\ \ -\infty < \tilde{z} < +\infty$. There is the split on the part of channel's lateral surface $\tilde{r} = R\,,\ \ -L \le \tilde{z} \le L$. The conducting fluid with constant velocity flow into the channel through this split. External magnetic field $\vec{B}^e = B_0 \vec{e}_z$ is parallel to the channel's axis. For the velocity's proection on $r-$ axis in Stokes and inductionless approximation we obtain the biharmonic equation

$$\left[L^2 + 2\frac{\partial^2}{\partial z^2}L + \frac{\partial^2}{\partial z^2}\left(\frac{\partial^2}{\partial z^2} - Ha^2\right)\right]V_r = 0, \tag{1}$$

where

$$L = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}. \tag{2}$$

The boundary conditions are:

$$r = 1,\ \ z \in (-L, L):\ V_r = -1;\ z \notin (-L, L),\ V_r = 0, \tag{3}$$

$$r = 1,\ \ -\infty < z < \infty:\ \frac{\partial}{\partial r}(rV_r) = 0. \tag{4}$$

The solution of the problem is obtained in form of integrals from the modified Bessel functions of the first kind. The asymptotic representation of the solution at Hartmann number $Ha \to \infty$ is obtaned. At $Ha = 0$ the problem (1), (3), (4) coincide with the problem from theory of elasticity for the function of stress in infinite cylinder (see [1], §144).

References

[1] S.P. Timoshenko, J.N. Goodier, *Theory of Elasticity*, McGrow-Hill, N.Y., 1970.

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