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Nonlinear Modulation of SH Waves in a Two Layered Plates and Formation of A.E.H.Love Waves

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ABSTRACT

In this work the propagation of small but finite amplitude shear horizontal (SH) waves in a hyperelastic two-layered plate of uniform thickness is considered. It is assumed that in a Cartesian reference frame (X,Y,Z) the top and bottom layers occupy the regions P_1,between the planes $Y = h_1$ and $Y = 0$, and P_2, between the planes $Y = 0$ and $Y = -h_2$, $(h_1 > 0,h_2 > 0)$ respectively. It is also assumed that the free boundaries $Y = h_1$ and $Y = -h_2$ are free of tractions, and the stresses and displacements are continuous at the interface $Y = 0$. The SH-waves are supposed to propagate along the positive X-axis and the displacement of a particle is denoted by u in P_1 and by v in P_2. u and v are only functions of X,Y and t,(time); because of the polarization of the waves. Assuming the constituent materials, homogeneous generalized neo-Hookean, first the governing equations and boundary conditions of the problem are written in terms of u and v. These equations constitute a nonlinear boundary value problem with the unknown functions u and v. Then the asymptotic solution of the problem characterizing the nonlinear self-modulation of a group of SH-waves centered around a wave number k and the corresponding frequency ω is considered. The method of multiple scales is employed [1] and u and v are expanded in asymptotic power series $u = \sum_{n=1}^{\infty} \epsilon^n u_n$ and $v = \sum_{n=1}^{\infty} \epsilon^n v_n$ in a small expansion parameter $\epsilon > 0$ which measures the degree of nonlinearity and the narrowness of the sideband width of the carrier wave number centered around a specific wave number. Employing the asymptotic expansions, a hierarchy of equations and boundary conditions are obtained. These perturbation problems are linear at each order and the first order problem is simply the elastic linear wave problem. The behavior of the first order solutions with respect to Y depend on the range of the phase velocities of c of the SH waves. With c_1 and c_2 denoting the linear shear wave velocities in the layers, two cases are to be considered. When $c_1 < c_2 < c$, it is found that u_1 and v_1 both depend on Y periodically. This case has been examined before in [2]. But if $c_1 < c < c_2$, the first order solutions are found to be

$$u_1 = \mathcal{A}_1(R_1 e^{ikpy} + R_2 e^{-ikpy})e^{i\Phi} + c.c. \quad , \quad v_1 = \mathcal{A}_1(R_3 e^{ik\nu y} + R_4 e^{-ik\nu y})e^{i\Phi} + c.c.$$

where $\Phi = kx_0 - \omega t_0$, $p = (c^2/c_1^2 - 1)^{\frac{1}{2}}$, $\nu = (1 - c^2/c_2^2)^{\frac{1}{2}}$, $x_n = \epsilon^n X, y = Y$, $t_n = \epsilon^n t$, R_m are some constants and $\mathcal{A}_1 = \mathcal{A}_1(x_1, x_2, t_1, t_2)$ is a complex amplitude function representing the first order slowly varying amplitude of the wave modulation. From the higher order perturbation problems it is found that $\mathcal{A}_1$ satisfies a nonlinear Schrodinger (NLS) equation. In the limit $h_2/h_1 \to \infty$, i.e. plate is covered by a very thin layer, the coefficients of the NLS equation obtained here approaches to the coefficients of the NLS equation obtained for the modulation of the surface SH-waves (Love waves) in [3].

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