3rd European Congress of Mathematics
Poster sessions

Section 14: Mathematical Physics Poster number 564

Electronics of receivers and number theory

Michel Planat*, Laboratoire de Physique et Métrologie des Oscillateurs, CNRS-UPR 3203, France.
Jacky Cresson, Université de Franche Comté, France.

ABSTRACT

We report on a mathematical understanding of superheterodyning discovered by Armstrong and Schottky in 1924 in the context of FM radio and now implemented in various communication systems, radars, spectrum analyzers... It is used to convert the information carried on an input oscillator of high frequancy f to a lower frequency F thanks to the use of a second input oscillator of frequency f_0.

Our new experiments [1] have found that, after low pass filtering, the resulting intermodulation frequencies $F = | pf_0 - qf |$ are controlled by *diophantine approximation* of $\nu = f/f_0$ with *spatial* and *time* resolution constraints. In order to take into account these fundamental properties of the frequency spectrum, we considered *continued fractions* (c-f) of ν and defined the space and time resolution effects.

A space resolution a_{max} (an integer) means that every number greater than a_{max} is identified with ∞. The action of $x \mapsto 1/x$ and $x \mapsto x + 1$ translates this constraint onto the c-f expansion. We obtain the set of c-f with *bounded partial quotients* which we denote $R_{a_{max}}$. It has a natural scaling structure which will be described in detail. The time resolution n_{max} (an integer) induces a finite number of inversions in the previous set : we have to truncate c-f to n_{max} terms. This induce natural *fuzzy* zones in the resolution space $R_{a_{max}}$.

The amplitude spectrum is much more complicated. Experiments show that it seems to be related to a frequency shift introduced in 1920 by Franel and Landau and as such to the uniform distribution of Farey points modulo 1. The asymptotic behaviour of this distribution is controlled via the Riemann hypothesis on the zeta function. Finally one can also explain the frequency noise at the receiver from the inverse zeta function at (or close to) the critical line which confirms the above approach.

Reference

[1] *in Noise, Oscillators and Algebraic Randomness: from Telecommunication Systems to Number Theory*, edited by M. Planat, Lecture Notes in Physics, Springer Verlag, 2000

Keywords: *Oscillators, finite continued fractions, Riemann zeta function*

Mathematics Subject Classification: *94, 11*

Contact Address: `planat@lpmo.edu`