Section 14: Mathematical Physics

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Inverse problem for the boundary condition with oblique derivative

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ABSTRACT_

Boundary-value problem with oblique derivative naturally arise in various transport problems in anisotropic media. We consider the charge transfer in anisotropic media whose anisotropy is due to the magnetic field. Thus we consider a metallic sample of the shape of a parallelepiped

 $V = \{x, y, z : 0 \le x \le a, 0 \le y \le c, 0 \le z \le b\}$ in a uniform magnetic field *B* parallel to the *y* axis. Suppose that on the parallelepiped's surface, in the planes perpendicular to the axes *z* and *y* the normal electrical currents $J_z = 0, J_y = 0$, and on both end-walls perpendicular to the *x* axis the potentials $\varphi_0(z), \varphi_1(z)$ are fixed. So we arrive at the following problem for electrical potential $\varphi(x, z)$:

$$\Delta \varphi = 0, \ \varphi|_{x=0} = \varphi_o(z), \ \varphi|_{x=a} = \varphi_1(z)$$
$$\left(\frac{\partial \varphi}{\partial z} + B\frac{\partial \varphi}{\partial x}\right)\Big|_{z=0,b} = 0$$

Let $\bar{\varphi}_{\delta} \stackrel{def}{=} \frac{1}{b} \int_{0}^{b} \varphi_{\delta}(z) dz$. Magnetic conductivity $\rho(B)$ is defined by

$$\rho(B) \stackrel{def}{=} \frac{a}{\bar{\varphi}_0 - \bar{\varphi}_1} \frac{1}{1 + B^2} \left(-\frac{1}{b} \int_0^b \frac{\partial \varphi}{\partial x}(x, z) dz + \frac{B}{b} \left(\varphi(x, b) - \varphi(x, 0) \right) \right)$$

and it is proved that conductivity does not depend on the point x [1]. The magnetoresistance $R(B) \stackrel{def}{=} 1/\rho(B)$ is the value which is measured in the experiments.

We consider the inverse problem: to calculate the boundary potentials $\varphi_{\delta}(z)$ through the magnetoresistance R(B) as a function of magnetic field. It is proved that for the |B| < 1 the R(B) is analytical function and we can reconstruct potential $\varphi_0(z)$ through the Taylor coefficients of the function R(B) and known another boundary function $\varphi_1(z)$.

Reference

"A problem with Directional Derivative in the Theory of Galvanomagnetic Effects". Yu. G. Gurevich, V. V. Kuchrenko, E. Ramírez de Arellano, Mathematical Notes, V. 65, No. 4, pp. 520–532.

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