PROJECT PROGRAMME

Network Title : Algebraic K-Theory, Linear Algebraic Groups and Related Structures

Network Short Title : K-Theory and Algebraic Groups

PART A – The Participants

The Contractor (Universität Bielefeld, Germany) will act jointly and severally towards the Commission with the following Associated Contractors to perform this Project Programme:

(Name of Legal Entity)	established in (name of state)
Universität Regensburg	Germany
Université de Franche Comté, Besançon	France
Université Paris 7-Denis Diderot	France
Université Louis Pasteur	France
Université catholique de Louvain	Belgium
Universitá di Genova	Italy
University College Dublin	Ireland
Odense Universitet	Denmark
Institute of Mathematics, National Academy of Sciences	Belarus
A. Razmadze Mathematical Institute	Georgia
St. Petersburg Branch of Steklov Mathematical Institute Russian Academy of Sciences	Russia

The Contractor and the Associated Contractors are hereinafter referred to jointly as “the Participants”.

PART B – The Joint Programme of Work

B.1 Research Objectives

It is the objective of this network project to perform advanced research in the fields of Algebraic K-Theory, Linear Algebraic Groups and Related Structures, by using methodological approaches from algebra and topology.
"Related Structures" are for example manifolds, schemes, quadratic or Hermitian forms, Azumaya algebras, Lie algebras, Brauer groups, and their associated homology and cohomology theories.
It is the main objective of this proposal to bring together the expertises of these distinct fields in order to promote their research significantly by mutual benefit, and, in particular, to attract young researchers at the postdoc level into this broad area by international exchange and intensive training.
Main research areas will be: Stable K-theory and Hochschild homology, K-theory of spaces, equivariant K-theory, algebraic models of spaces, Leibniz homology, controlled K-theory, K-theory of varieties, linear algebraic groups over arbitrary fields, projective homogeneous varieties, Hasse principles for principal homogeneous spaces and other objects, splitting varieties, quadratic forms over arbitrary fields, trace forms, Azumaya algebras, algebras with involution, Chow groups, Brauer groups, R-equivalence and birational questions, toric varieties, cohomology ring of finite group schemes.

B.2 Methodological Approach and Work Plan

This project combines areas which have developed their own "mathematical culture", like algebraic K-theory, linear algebraic groups, quadratic and Hermitian forms, Brauer groups, Azumaya algebras and others.
As history has shown that these fields can influence each other in a very fruitful way, it is the incentive of this proposal to bring scientists of these various branches together for useful cooperation.
Since the scientific resources of the single EC countries are limited, it is intended to overcome these limitations by extensive exchange of scientists, in particular, of postdoc scientists.
Postdocs should have the possibility to work for 12 to 18 months, say, in a team in a different country in order to be trained by the host team's expertises. Since individual research or research in small groups is the appropriate approach in the field of mathematics, and since all the teams involved in this project are guided by internationally very well recommended end experienced researchers, who have significant experiences in the training and the educating of young scientists, it is guaranteed that this methodological approach is appropriate and consistent with the objectives of this proposal.
The work in small teams is accompanied by a reasonably chosen number of conferences and workshops, which allow a regular exchange of new ideas in the various scientific areas of this project.
Since all the fields mentioned in the proposal are represented in the teams of this proposal by internationally leading experts, it is also obvious that the proposed network integrates complementary techniques in an effective manner.

Workplan by Research Topics, as performed at the various participants

Project milestones are marked by '!', midterm assessment criteria are marked by a '*'.

Team 1: Bielefeld

- • Stable K-theory and topological Hochschild homology of A_∞ rings
- * The canonical involution on the algebraic K-theory of spaces
- * Excellence properties of semisimple groups
- * Periodicity of twist types of orthogonal groups
- ! Splitting properties of anisotropic algebraic groups over arbitrary fields, Azumaya algebras, quadratic forms

Team 2: Regensburg

- * A formally real analogue of Grothendieck's theorem on noncommutative H^2
- ! Hasse principles for homogeneous spaces over fields with vcd=2
- ! Principal homogeneous spaces with prescribed local invariants
- ! Composition and Jordan algebras over certain schemes
- ! Characteristic numbers of splitting varieties of symbols in Galois cohomology
- ! H^3 mod 5 invariant for E_8 and H^3 mod 4 invariant for E_7

Team 3: Besançon

- * Progress in any of the following could serve as a midterm assessment criterion.
- • Linear algebraic groups and their Galois cohomology
- • Cohomological invariants of linear algebraic groups
- • Trace forms of Galois algebras
- • Quadratic forms over arbitrary fields
- • Local–global questions for bilinear forms

Team 4: Paris

- • Algebraic models of spaces (related to team 1, 3, 5, 9)
- • Quadratic forms, Lichtenbaum-Quillen conjecture, Galois cohomology (related to team 1, 2, 3, 5, 8)
- • Investigation of the category of Feynman diagrams (related to team 5, 9)
- • Homotopy theory of vectoriel functors (related to team 1, 5, 9)
- • Cyclic Hochschild homology of the ring of invariants of the Weyl algebra under action of a finite group, cyclic homology of commutative algebras in characteristic p (related to team 1, 5, 9)
- • Equivariant K-theory of proper actions of infinite discrete groups (related to team 1, 5, 9)
- • MacLane Homology (related to team 1, 5).
- ! Homotopy theory of algebraic varieties, relations with the theory of mixed motives, Grothendieck duality, etc...
- ! Relation between p-adic regulators and "ultrametric" K-theory

Team 5: Strasbourg

- $\ast$ Leibniz homology for groups, non-commutative K-theory
- • Non-commutative cohomology of discrete groups
- ! Borel regulators, polylogarithms, Lichtenbaum conjecture
- • Operations on cohomology and homotopy theory

Team 6: Louvain-La-Neuve

- • Investigation of trace forms

! Classification of central simple algebras with involutions over fields where quadratic forms are classified by the classical invariants.
! Description of Witt kernels
* Completion of a monograph on algebras with involution.
* Description of an algebra with involution which parametrizes the Witt kernel of a finite extension.

Team 7: Genova This team deals mainly with application of K-theory to

- algebraic geometry: Chow groups, Bloch's conjecture for surfaces, topological invariants of real algebraic varieties, in particular in the singular case.
- algebraic groups: normal subgroups in Chevalley groups, representations of rational subgroups over rings,

! Chevalley groups over rings

- Commutative algebra and computer algebra.

Team 8: Dublin

- The study of trace forms and their generalizations on central simple algebras.
- The study of quadratic- form-theoretic invariants for involutions on central simple algebras. (these topics will have some overlap with work of the Belgian team and the Besançon team)
- The applications of algebraic K-theory and algebraic L-theory to the computation of knot cobordism groups.

Team 9: Odense

- Computing K-theory (of rings or spaces) by homotopy theoretic means
- K-theory and homology of Lie groups considered as discrete groups
- Controlled K-theory and its applications to problems of a geometric origin, especially the Novikov Conjecture

! Homotopy of $\mathrm{TC}(X \to Z[\pi_1 X])$ in terms of more accessible constructions in homotopy theory
! Progress towards the verification of the Lichtenbaum - Quillen conjecture for local number rings (or equivalently, calculate $\mathrm{TC}(R)$)
! Obtaining torsion information about the space of diffeomorphisms of discs and other simply-connected smooth manifolds

Team 10: Minsk

- Investigation of the 2-torsion part of Brauer groups of local and global curves and varieties
- R-equivalence and birationality questions for algebraic groups
- Investigation of group 0-cycles modulo rational equivalence for surfaces

Team 11: Tbilisi

- Investigation of topological Hochschild homology
- Use of toric varieties to study polyhedral linear groups
- Excision, regularity and relationship with topological stability for algebraic K-theory
- Cohomological investigation of Lie and associative algebras
- Calculus of polynomial functors and related topological and categorical structures

Team 12: St. Petersburg

! Study the cohomology ring of a finite group scheme over a field k
! The relationship between G-equivariant K'-theory and the K-theory of any variety, where G is a simply connected algebraic group
! Investigation of 14-dimensional quadratic forms

B.3 Schedule and Milestones

Some of the project milestones have already been indicated in the preceding section. The following Conferences and the results presented there are considered as important milestones of the Project.
The time is given in terms of the number of months elapsed from the start of the project.

7 **Summer school** concerning K-theory and its applications to geometry and topology in Trento
8 **Conference** "Linear algebraic groups and related structures" in Bielefeld
8 **Conference** "Topology and Geometry" in Aarhus,
The two above conferences are satellite conferences to the International Congress of Mathematicians.
15 **Workshop** "Generic splitting varieties", in Regensburg, Spring 1999
19 **Conference** "Algebraic K-theory and related structures" in Paris
19 **Euroconference** "Quadratic forms and Topology" in Dublin
27 **Conference** "Linear algebraic groups and related structures" at Bielefeld
At this conference, the **mid term evaluation** of the project should be performed.
Further coordinated workshops on actual topics, among them:
31 **Summer school** in Strasbourg or Marseille-Luminy on the techniques of derived functors over "non-additive bimodules" : "From K-theory to cohomology"
34 "Thanksgiving **Workshop** on Controlled Topology, Geometry and Algebra"
43 "**Workshop** on various topics of the project". At this workshop, particularly the young researchers sponsored by this project should have the opportunity to present their results.

B.4 Research Effort of the Participants

Part. No.	Team of University/Institute:	is composed by members of Universities/Institutes
1	Bielefeld	Bielefeld, Osnabrück
2	Regensburg	Regensburg
3	Besançon	Besançon
4	Paris 7	Paris 7, Paris 13, Nantes, Ecole Polytechnique
5	Strasbourg	Strasbourg, Montpellier, Nice, Valencienne
6	Louvain-la-Neuve	Louvain-la-Neuve, Ghent
7	Genova	Genova, Napoli, Milano, Torino
8	Dublin	Dublin, Edinburgh
9	Odense	Odense, Aarhus, Oslo
10	Minsk	Minsk
11	Tbilisi	Tbilisi
12	St. Petersburg	Steklov Inst. and University St. Petersburg

Participant	Professional Research Effort (man-months) on the Network Project			Number of individual researchers involved
	by young researchers financed by the contract (a)	by researchers financed from other sources (b)	Total research effort (a+b)	
1.	30	96	126	10
2.	30	48	78	6
3.	38	32	70	6
4.	36	198	234	11
5.	48	156	204	13
6.	30	36	66	5
7.	36	150	186	13
8.	38	52	90	8
9.	30	126	156	11
10.		48	48	4
11.		108	108	9
12.		60	60	5
Total	316	1110	1426	101

B.5 Network Organisation and Management

The typical technique to coordinate the network will be by electronic means:
All coordinators and most individual participants have access to electronic mail, this will be the typical tool of communication providing good communication between the research teams. Here is the list of the email addresses of the Network Coordinator (NC) and the local coordinators (LC).

Part.	Fct.	Name	Email Address
1.	NC	Ulf Rehmann	rehmann@mathematik.uni-bielefeld.de
2.	LC	Claus Scheiderer	Claus.Scheiderer@mathematik.uni-regensburg.de
3.	LC	Eva Bayer-Fluckiger	bayer@vega.univ-fcomte.fr
4.	LC	Max Karoubi	karoubi@math.jussieu.fr
5.	LC	Jean-Louis Loday	loday@math.u-strasbg.fr
6.	LC	Jean-Pierre Tignol	tignol@agel.ucl.ac.be
7.	LC	Claudio Pedrini	pedrini@dima.unige.it
8.	LC	David Lewis	dwlewis@ollamh.ucd.ie
9.	LC	Hans J. Munkholm	hjm@imada.ou.dk
10.	LC	Viacheslav Yanchevskii	imanb@imanb.belpak.minsk.by
11.	LC	Teimuraz Pirashvili	pira@imath.acnet.ge
12.	LC	Andrei Suslin	suslin@pdmi.ras.ru

In order to quickly distribute research results, a World Wide Web server will be set up by the network coordinator at U. Bielefeld. This will, besides electronic mail, be used to disseminate informations about important events in the project like seminars, workshops, conferences, and will provide an appropriate tool for monitoring and reporting progress.

The following matrix describes the cooperation linkages between the participants which already exist at project start.

Participant	No.	cooperates with participant											
		1	2	3	4	5	6	7	8	9	10	11	12
Bielefeld	1		+	+	+	+	+	+		+	+	+	+
Regensburg	2	+		+	+	+	+	+					+
Besançon	3	+	+		+		+		+		+		+
Paris	4	+	+	+		+		+		+		+	+
Strasbourg	5	+	+		+				+	+		+	
Louvain-la-Neuve	6	+	+	+					+		+		+
Genova	7	+	+		+				+				
Dublin	8			+		+	+	+		+			
Odense	9	+			+	+			+				
Minsk	10	+		+			+						+
Tbilisi	11	+			+	+				+			
St.Petersburg	12	+	+	+	+		+				+		

B.6 Connections to Industry

No connections to industry are given, since this project is research in pure mathematics.

PART C – Training

C.1 Employment of Young Researchers

Parti-cipant	Young Researchers Financed by Contract (man-months)			Scientific specialties
	Pre-doctoral researchers (a)	Post-doctoral researchers (b)	Total (a+b)	
1.		30	30	Algebra/Topology M42/43
2.		30	30	Algebra M42
3.		38	38	Algebra M42
4.		36	36	Algebra/Topology M42/43
5.	12	36	48	Algebra/Topology M42/43
6.		30	30	Algebra M42
7.		36	36	Algebra M42
8.		38	38	Algebra M42
9.		30	30	Algebra/Topology M42/43
10.				Algebra M42
11.				Algebra M42
12.				Algebra M42
Total	12	304	316	

The total of 316 man-months (column a+b) is the minimum number of man-months of young researchers whose employment will be financed by the contract. The breakdown between a and b is indicative.

Vacancies will be published on the internet at various places:

– web pages of the network,
– web pages kept at Brussels,
– by broadcasting them to other mathematical institutes via flyers and via the national network structures.

The typical appointment length is expected to be in the range of 8 – 16 months.

C.2 Training Programme

This project offers an exchange program consisting of 316 pre- and post-doctoral fellowships, which shall be supported by additional funds for the organization of *accompanying conferences, summer schools and workshops*, and for mutual short term visits. Every team of this network is well prepared to train young scientists from other European countries on the occasion of visits and secondments, participation on conferences, workshops and summer schools.
All network teams are guided by experienced senior scientists, who are among the leading in there fields, working in well established groups, usually with many scientific guests, in mathematics departments with elaborated training programmes including *lectures, seminars, workshops*, on the doctoral and post-doctoral level. Hence visiting pre- and post-doctorals and other staff working on the project are guaranteed to receive optimal training in an academic environment.

Special measures to be taken: All teams are – as university departments of mathematics – experienced in offering high level research seminars on various fields of mathematics, which is their usual standard.
Moreover, *dedicated conferences, summer schools and workshops* will be organized by various teams, which will take advantage of the international character of the network by inviting leading experts from other teams, and which will – in particular – give the young researchers the opportunity to participate and to present their results achieved under this programme.