Algebraic K-Theory, Linear Algebraic Groups and Related Structures

1. Research Topic

The fields of "algebraic K-theory" and of "linear algebraic groups" are very vivid and very competitive research areas.
During the last few years, several important results have been found which have stimulated research in these fields considerably.
For example, methods in K-theory made it possible to prove the so called "index reduction theorems" by various authors, among them Schofield/v.d.Bergh (1994) and Merkurjev/Panin/Wadsworth (1996), which are of considerable impact on the theory of algebraic groups as well as on related structures like quadratic forms or Azumaya algebras without and with involution, and which allow to develop tools in order to study anisotropic semisimple groups over arbitrary fields, an area which hitherto, even after a hundred years of research about semisimple groups, appeared almost unaccessible.
Another important actual result is the recent confirmation of the long standing Milnor conjecture by Voevodsky (1996), which relates Milnor K-theory with certain étale cohomology groups, and which has important consequences, e.g., for the theory of quadratic forms and hence for orthogonal groups.
These examples show that there is a fruitful connection between algebraic K-theory on the one hand and linear algebraic groups on the other, and that it makes sense right now to set up a research project bringing these two fields together in order to promote them both on the basis of the newly developed results.

2. Project Objectives and 3. Scientific Originality of the Project

Algebraic K-theory is a quite interdisciplinary area within mathematics. Here are some prominent aspects of this feature. First, the construction of higher algebraic K-theory, due to Quillen, uses the elaborate machinery of algebraic topology in an essential way; in fact, the very definition of the higher K-groups is in terms of homotopy groups in the sense of algebraic topology—no other definition of the higher K-groups is known to this date. Second, algebraic K-theory has found applications to the topology of manifolds and their automorphisms; keywords are: pseudoisotopy theory, and the algebraic K-theory of topological spaces of Waldhausen. Third, the latter theory necessitated a technical development which has repercussions to algebraic K-theory proper; specifically it has enabled Thomason to develop higher K-theory for schemes with far reaching applications in algebraic geometry. Fourth, but probably not finally, the theory of the trace map which into higher algebraic K-theory was introduced by Dennis has found a substantial extension to a theory involving the so-called topological Hochschild homology theory. A variant of this, the topological cyclic homology, has permitted the solution, by Bökstedt, Hsiang, Madsen, of the K-theory analog of the Novikov conjecture; and, more recently, thanks to the combined work of McCarthy, Dundas, Bökstedt, Madsen the long-sought computation of the algebraic K-theory of the ring of the p-adic integers. Topological Hochschild and cyclic homology are the analogues of the corresponding classical notions for A_∞ and E_∞ ring spectra and their module spectra. The algebra of these spectra has turned out to be of great importance for investigations in stable homotopy theory.

THE THEORY OF LINEAR ALGEBRAIC GROUPS has its origins in the profound work of Killing, who found the classification of semisimple Lie groups in 1888. Since then many prominent mathematician, among them Cartan, Weyl, Chevalley, Borel, Springer, Tits have elaborated a theory of semisimple algebraic groups over *arbitrary fields*, which describes the internal structure of those groups by means of so called “BN-pairs” or “Tits systems” up to their respective “anisotropic kernels”, which are “anisotropic” semisimple groups, characterized by the fact that, over their field of definition, they do not contain unipotent elements. These groups still are, after a whole century of intense research, far from being understood. However, there are newly developed tools like the already mentioned index reduction theorems, but also like the theory of generic splitting of reductive groups (Kersten/Rehmann, 1994), or the Galois cohomological invariants (Rost, Serre), which give substantial hope to gain a better understanding of anisotropic groups. Other promising and important progress has recently been made in these fields by Bayer-Fluckiger and Parimala (1995): They proved a 30 year-old conjecture of Serre for classical (and some exceptional) groups concerning the vanishing of the first Galois cohomology set of semi-simple, simply connected groups over fields of cohomological dimension 2. This generalizes classical “local-global” results.
RELATED STRUCTURES in the context of these theories are, for example, modules over rings, quadratic and Hermitian forms, Witt groups and rings, Lie algebras, Azumaya algebras, possibly with involutions, Brauer groups, Jordan algebras, projective homogeneous and toric varieties. Most of these have developed their own mathematical culture. The common feature of all these algebraic structures is that their automorphism groups are reductive groups, and many aspects of these distinct theories can be generalized and unified by studying these automorphism groups.

Ever since, these areas of research have influenced each other in a very fruitful way. Let us mention the investigations for the congruence subgroup problem which solved difficult questions concerning the subgroup structure of certain classical and more general linear groups by studying questions which are typical for algebraic K-theory like stability questions. It was a result of Merkurjev about quadratic forms which laid the foundations for the very influential and famous K-theoretic “Theorem of Merkurjev and Suslin” about the norm residue symbol over arbitrary fields proved in 1981. The determination of various (co-)homology groups of linear groups were transformed into questions about certain ‘K-groups’. K-theoretic methods in turn allowed Merkurjev recently to disprove a longstanding conjecture of Kaplansky about the so called u-invariant of quadratic forms.

It is the main objective of this proposal to bring together the expertises of these fields in order to promote their research significantly by mutual benefit, and, in particular, to attract young researchers at the postdoc level into this broad area by international exchange and intensive training.

4. Research Method

This project combines areas which have developed their own "mathematical culture", like algebraic K-theory, linear algebraic groups, quadratic and Hermitian forms, Brauer groups, Azumaya algebras and others.
As history has shown that these fields can influence each other in a very fruitful way, it is the incentive of this proposal to bring scientists of these various branches together for useful cooperation.
Since the scientific resources of the single EC countries are limited, it is intended to overcome these limitations by extensive exchange of scientists, in particular, of postdoc scientists.
Postdocs should have the possibility to work for 12 to 18 months, say, in a team in a different country in order to be trained by the host team's expertises. Since individual research or research in small groups is the appropriate in the field of mathematics, and since all the teams involved in this project are guided by internationally very well recommended end experienced researchers, who have significant experiences in the training and the educating of young scientists it is guaranteed that this methodological approach is appropriate and consistent with the objectives of this proposal.
The work in small teams is accompanied by a reasonably chosen number of conferences and workshops, which allow a regular exchange of new ideas in the various scientific areas of this project.
Since all the fields mentioned in the proposal are represented in the teams of this proposal by internationally leading experts, it is also obvious that the proposed network integrates complementary techniques in an effective manner.

5. Workplan

Project milestones are marked by '!', midterm assessment criteria are marked by a '*'.

Team 1: Bielefeld

- • Stable K-theory and topological Hochschild homology of A_∞ rings (Schwänzl, Waldhausen, Vogt, team 1, and Staffelt, USA)
- * The canonical involution on the algebraic K-theory of spaces (Klein, Vogell, Waldhausen, team 1, and Williams, USA)
- * Excellence properties of semisimple groups (Kersten, Rehmann).
- * Periodicity of twist types of orthogonal groups (Rehmann).
- ! Splitting properties of anisotropic algebraic groups over arbitrary fields, Azumaya algebras, quadratic forms (I. Kersten, U. Rehmann)

Team 2: Regensburg

- * A formally real analogue of Grothendieck's theorem on noncommutative H^2 (Scheiderer)
- ! Hasse principles for homogeneous spaces over fields with vcd=2 (Scheiderer)
- ! Principal homogeneous spaces with prescribed local invariants (Scheiderer)
- ! Composition and Jordan algebras over certain schemes (Pumplün)
- ! Characteristic numbers of splitting varieties of symbols in Galois cohomology (Rost).
- ! H^3 mod 5 invariant for E_8 and H^3 mod 4 invariant for E_7 (Rost).

Team 3: Besançon

- * Progress in any of the following could serve as a midterm assessment criterion.
- • Linear algebraic groups and their Galois cohomology
- • Cohomological invariants of linear algebraic groups
- • Trace forms of Galois algebras
- • Quadratic forms over arbitrary fields
- • Local–global questions for bilinear forms

Team 4: Paris

- • Algebraic models of spaces (Karoubi) (related to team 1, 3, 5, 9)
- • Investigation of the category of Feynman diagrams (Vogel) (related to team 5, 9)
- • Homotopy theory of vectoriel functors (Franjou) (related to team 1, 5, 9)
- • Cyclic Hochschild homology of the ring of invariants of the Weyl algebra under action of a finite group, cyclic homology of commutative algebras in characteristic p (Vigue) (related to team 1, 5, 9)
- • Equivariant K-theory of proper actions of infinite discrete groups (Oliver) (related to team 1, 5, 9)
- • Homologie de MacLane, plus specifiquement etude des extensions entre foncteurs simples dans la categorie F des foncteurs de la categorie des espaces vectoriels sur un corps fini dans elle même. (Schwartz) (related to team 1, 5).
- ! Homotopy theory of algebraic varieties, relations with the theory of mixed motives, Grothendieck duality, etc... (Morel)
- ! Relation between p-adic regulators and "ultrametric" K-theory (Karoubi).

Team 5: Strasbourg The construction of a Leibniz homology theory for groups (and hence for any topological space) is a milestone in the current project. It seems that a complete solution requires the understanding of non-Abelian cohomology of discrete groups (D. Guin). The expectation is the discovery of a "non-Abelian K-theory" (J.L. Loday) which would help a lot to understand motivic cohomology, the Beilinson-Soule conjectures and the Borel regulators made of polylogarithms (J.-L. Cathelineau).

Team 6: Louvain-La-Neuve

- • Investigation of trace forms (Tignol)
- ! Classification of central simple algebras with involutions over fields where quadratic forms are classified by the classical invariants.
- ! Description of Witt kernels
- * Completion of a joint monograph on algebras with involution, with M. Knus, A. Merkurjev and M. Rost.
- * Description of an algebra with involution which parametrizes the Witt kernel of a finite extension.

Team 7: Genova This team deals mainly with application of K-theory to

- • algebraic geometry: Chow groups, Bloch's conjecture for surfaces, topological invariants of real algebraic varieties, in particular in the singular case.
- • algebraic groups: normal subgroups in Chevalley groups, representations of rational subgroups over rings,
- ! Chevalley groups over rings
- • Commutative algebra and computer algebra.

Team 8: Dublin

- • The study of trace forms and their generalizations on central simple algebras.
- • The study of quadratic- form-theoretic invariants for involutions on central simple algebras. (these topics will have some overlap with work of the Belgian team and the Besançon team)
- • The applications of algebraic K-theory and algebraic L-theory to the computation of knot cobordism groups.

Team 9: Odense

- • Computing K-theory (of rings or spaces) by homotopy theoretic means (Bökstedt, Madsen)
- • K-theory and homology of Lie groups considered as discrete groups (Dupont).
- • Controlled K-theory and its applications to problems of a geometric origin, especially the Novikov Conjecture (Pedersen, Munkholm).
- ! Homotopy of $\mathrm{TC}(X \to Z[\pi_1 X])$ in terms of more accessible constructions in homotopy theory (Bökstedt, Madsen).
- ! Progress towards the verification of the Lichtenbaum - Quillen conjecture for local number rings (or equivalently, calculate $\mathrm{TC}(R)$) (Bökstedt, Madsen).
- ! Obtaining torsion information about the space of diffeomorphisms of discs and other simply-connected smooth manifolds (Rognes, Jahren).

Team 10: Minsk

- • Investigation of the 2-torsion part of Brauer groups of local curves (Yanchevskii)
- • R-equivalence and rationality questions for algebraic groups (Chernousov)

Team 11: Tbilisi

- • Investigation of topological Hochschild homology
- • Use of toric varieties to study polyhedral linear groups.
- • Cohomological investigation of Lie and associative algebras

Team 12: St. Petersburg

- ! Study the cohomology ring of a finite group scheme over a field k (Suslin)
- ! The relationship between G-equivariant K'-theory and the K-theory of any variety, where G is a simply connected algebraic group (Merkurev, Panin).
- ! Investigation of 14-dimensional quadratic forms (Izhboldin, Karpenko).

Schedule: Conferences and workshops

1997 Algebraic K-theory **Workshop/Symposium** in Trieste, organized by H. Bass (Columbia), A. O. Kuku (ICTP), C. Pedrini (Genova), September 1-19, 1997.

1998 **Conference** "Linear algebraic groups and related structures" in Bielefeld, organized by I. Kersten (Bielefeld), A. Merkurjev (St. Petersburg), U. Rehmann (Bielefeld), August 10 – 14, 1998

1998 **Conference** "Topology and Geometry" in Aarhus, organized by K. Grove (Maryland), I. Madsen (Aarhus) and E. K. Pedersen (Odense), August 10 – 16, 1998

The two above conferences are satellite conferences to the International Congress of Mathematicians.

1998 **Summer school** concerning K-theory and its applications to geometry and topology in Trento, organized by C. Pedrini (50-60 participants).

1999 **Workshop** "Generic splitting varieties", in Regensburg, Spring 1999, organized by M. Rost, Regensburg.

1999 **Euroconference** "Quadratic forms and related topics" in Dublin, organized by D. Lewis and A. Ranicki.

1999 **Conference** "Algebraic K-theory and related structures" in Paris, organized by M. Karoubi, July 1999.

Further coordinated workshops on actual topics, among them:

- Munkholm plans to repeat his earlier "Thanksgiving **Workshops** on Controlled Topology, Geometry and Algebra" (1990, 1994).
- **Summer school** in Strasbourg or Marseille-Luminy on the techniques of derived functors over "non-additive bimodules" : "From K-theory to cohomology", organized by J.-L. Loday.

The conferences should be considered as important milestones of the project. Depending on the actual beginning of this project, the results presented on the conferences in 1998 and/or 1999 could serve as criteria for the midterm evaluation of the project. The following **table** indicates the size **of the professional research effort** (in man-months), differentiating between young visiting researchers (YVR) and researchers financed from other sources (RFO) for the EC teams.

EC-Countries:		Teams:	YVR	RFO
	1.	Bielefeld	30	84
	2.	Regensburg	30	48
	3.	Besançon	30	32
	4.	Paris	36	180
	5.	Strasbourg	36	84
	6.	Louvain-la-Neuve	30	36
	7.	Genova	36	150
	8.	Dublin	36	42
	9.	Odense	30	135
		Total :	294	791

The teams from Third Countries contribute as follows (RFO only): Minsk 48, Tbilisi 108, St. Petersburg 60, total 216.

6. Collective Experience of the Research Teams

6.1 Bielefeld: *Description of recent research activities of the team:* Algebraic K-theory of spaces, $A_\infty-$ and E_∞ rings and spectra of rings, Hochschild homology and cohomology, Algebra and homological algebra over ring spectra, Hermitian K-theory and its relation to the geometry of manifolds (Vogt, Waldhausen), Generic splitting of algebraic groups, Splitting patterns and excellence properties of anisotropic semisimple groups and of quadratic forms over arbitrary fields, twist types of semisimple algebraic groups, quadratic and Hermitian forms, Brauer groups (Kersten, Rehmann)
Linkages: Regensburg (Rost), Besançon (Bayer), Paris (Kahn), Strasbourg (Loday), Louvain-la-Neuve (Tignol), Odense (Bökstedt, Madsen), Minsk (Yanchevskii), Tbilisi (Pirashvili), St. Petersburg (Merkurjev, Panin)
Key scientific staff (man-months in ()): Abels (12), Kersten (12), Klein (12), Rehmann (12), Waldhausen (12) (all Bielefeld), Plümer (12), Schwänzl (12), Vogt (12) (all Osnabrück).
Publications:
– I. Kersten and U. Rehmann, Generic splitting of reductive groups, *Tôhoku Math. J.* **46** (1994), 35-70.
– Z. Fiedorowicz, T. Pirashvili, R. Schwänzl, R. Vogt, F. Waldhausen, MacLane homology and topological Hochschild homology, Math. Ann. **303** (1995), 149-164.

6.2 Regensburg: *Description of recent research activities of the team:*
S. Pumplün has worked over composition algebras over open dense subschemes of curves of genus zero. There is a close connection between composition and Jordan algebras, and exceptional simple algebraic groups. In particular, this connection exists between octonions and groups of type G_2. It is planned to investigate these algebras over certain schemes, using recent results on quadratic forms and modules.
C. Scheiderer: Hasse principles for the existence of rational points in homogeneous spaces have recently gained some interest in the context of formally real fields of small virtual cohomological dimension, through work of Colliot-Thélène, Bayer-Parimala and Scheiderer. Research projects are: (1) Hasse principles for certain homogeneous spaces over formally real fields k with $\mathrm{vcd}(k) = 2$. (2) Existence of principal homogeneous spaces under linear algebraic groups over formally real fields with prescribed local invariants. (3) Proof of a formally real analogue of Grothendieck's theorem on noncommutative H^2. (4) Proof of Serre's Conjecture II for exceptional groups, and of its formally real analogue.
M. Rost has worked on Chow groups with coefficients, algebras with involutions, on composition algebras, on many aspects of quadratic forms and on various questions concerning algebraic K-theory.
1. Cohomology of splitting varieties: Concerning the Bloch-Kato conjecture an important role is played by the generic splitting varieties of symbols in Milnor's K-theory K_n^{M}/p. This has been clear since the work of A. A. Merkurjev and A. S. Suslin in 1982. The recent work of V. Voevodsky has put special attention to two specific problems: a) Find splitting varieties X of dimension $p^{n-1}-1$ which are generators of the cobordism ring (localized in p). This amounts to the computation of a certain characteristic number. b) Show that the norm map $A_0(X, K_1) \to K_1F$ on zero cycles with coefficients in K_1 is injective. Problem a) introduces cobordism theory into the field. This is a rather

new topic and opens a wide area of research. Problem b) has been considered for some years. Nevertheless the new attention to it promises a fruitful development; already in the last few month there were new results on classical topics like the chain-lemma for algebras and exceptional Jordan algebras, octave planes, etc. Also the theme of compactification of simply connected groups has to be mentioned here.

2. Cohomological invariants: In recent years there has been some new developments concerning the Galois cohomology set $H^1(F,G)$, with strong influence of J.-P. Serre. E.g., there is the H^3-invariant and the proof of Serre's conjecture II for classical groups. Still there are a lot of questions which ask for specific research: Very general problems like the overall understanding of the "space" of cohomological invariants (for a fixed group G, but also with respect to degenerations—Bruhat-Tits-theory), but also more specific problems like Serre's conjecture II for exceptional groups (starting with triality-D_4) or just the understanding of the mod4 invariant for E_7.

Linkages: Bielefeld (Kersten, Rehmann), Besançon (Bayer), Paris (Kahn), Louvain-la-Neuve (Tignol), St. Petersburg (Merkurjev, Panin)

An important book project involves 3 teams: M.-A. Knus, A. S. Merkurjev, M. Rost, J.-P. Tignol, The Book of Involutions, in preparation.

Key scientific staff (man-months in ()): Knebusch (12), Pumplün (12), Rost (12), Scheiderer (12)

Publications:

– M. Rost: Chow groups with coefficients. Documenta Math. 1, 319-394 (1996)

– C. Scheiderer: Hasse principles and approximation theorems for homogeneous spaces over fields of virtual cohomological dimension one. Invent. math. 125, 307-365 (1996)

6.3 Besançon: *Description of recent research activities of the team:*

The main focus of the research of this team concerns linear algebraic groups and their Galois cohomology. In particular, E. Bayer–Fluckiger and R. Parimala have recently proved a conjecture of Serre made in 1962 concerning the vanishing of the first Galois cohomology set $H^1(k,G)$ when k is a field of cohomological dimension 2 and G a semi–simple, simply connected group of classical type. They also proved a recent conjecture of Colliot–Thélène concerning the Hasse principle for fields of virtual cohomological dimension 2. Other research results concern cohomological invariants of linear algebraic groups, as trace forms of Galois algebras, quadratic forms over arbitrary fields, as well as local–global questions for bilinear forms.

The team has several research students, at the doctoral or post-doctoral level. Some of them come to Besançon in the framework of a cooperation program with Minsk, Moskow and St. Petersburg. Each week, the mathematicians of the team meet in a seminar (joint with other algebraists and number theorists in Besançon) as well as in 3 "learning seminars" on algebraic K–theory, group cohomology and algebraic geometry. The students also take part in national and international workshops. For instance, Besançon was the coordinating partner in a "Euroconferences" series on K–theory and linear algebraic groups.

Linkages: The team has intensive cooperation with Bielefeld (Kersten, Rehmann), Dublin (Lewis), Louvain-la-Neuve (Tignol, van Geel), Regensburg (Rost, Scheiderer), Paris 7 (Kahn, Karoubi), Minsk (Yanchevskii, Chernousov), St. Petersburg (Merkurjev, Panin). In this cooperation, the exchange of doctoral and postdoctoral students has a central role.

Key scientific staff (man-months in ()): E. Bayer (12), A. Cortella (10), D. Hoffmann (10).
Significant recent publications:
– E. Bayer–Fluckiger, J-P. Serre, Torsions quadratiques et bases normales autoduales, *Amer. J. Math.* **116** (1994), 1–64.
– E. Bayer–Fluckiger, R. Parimala, Galois cohomology of linear algebraic groups over fields of cohomological dimension ≤ 2, *Invent. Math.* **122** (1995) 195–229.

6.4 Paris: *Description of recent research activities of the team:* Cyclic homology, algebraic models of spaces (Karoubi), Vassiliev theory, applications to node theory and to the category of modules over Lie algebras (Vogel), MacLane cohomology and cohomology of linear groups (Franjou), Hochschild homology, cyclic homology (Vigue), equivariant K-theory (Oliver), Geometrie d'Arakelov, theorie des motifs (Soulé), MacLane homology (Schwartz).
Linkages: Strasbourg (Loday), Besançon (Bayer), Bielefeld (Kersten, Rehmann, Waldhausen, Vogt), Odense/Aarhus (Madsen, Munkholm, Bökstedt), Tbilisi (Pirashvili), Genova (Pedrini).
Key scientific staff (man-months in ()): Franjou (18), B. Kahn (36) , M. Karoubi (18), C. Soulé (36), Morel (36), R. Oliver (18), Schwartz (18), Vogel (18).
Recent Publications:
– Bruno Kahn. Applications of weight-two motivic cohomology. Documenta Math. 1 (1996) 395-416.
– Max Karoubi. K-théorie multiplicative. Proceedings of the 1996 Waterloo Conference in cyclic homology.

6.5 Strasbourg: *Description of recent research activities of the team:* All aspects of algebraic K-theory and its additive analogue: cyclic homology with emphasis on non-abelian techniques like polynomial functors (relating K-theory and Topological Hochschild Homology), operads, non-abelian cohomology of groups, Leibniz algebras
Linkages: Tbilisi (Pirashvili, Inassaridze), Odense/Aarhus (Bökstedt, Dupont, Madsen), Regensburg (Rost), Paris (Karoubi, Schwartz, Kahn), Bielefeld (Vogt, Waldhausen, Rehmann).
Key scientific staff (man-months in ()): Christian Kassel (12), Jean-Louis Loday (12) (both Strasbourg), Claude Cibils (12) , Daniel Guin (12) (both Montpellier), Jean-Louis Cathelineau (12) (Nice), Francois Goichot (12), Manfred Hartl (12) (both Valencienne).
Recent Publications:
– Cathelineau, Jean-Louis Homologie du groupe linéaire et polylogarithmes, Séminaire Bourbaki, Vol. 1992/93. Astérisque No. 216 (1993), Exp. No. 7= 72, 5, 311–341.
– Loday, Jean-Louis Cyclic homology. Grundlehren der Math. Wiss., 301. Springer-Verlag, Berlin, 1992.

6.6 Louvain-la-Neuve: *Description of recent research activities of the team:* The Belgian team has been concentrating on the algebraic theory of quadratic forms from the function field and from the involution point of view for several years. The idea of considering central simple algebras as twisted forms of quadrics has been developed in Louvain-la-Neuve since 1991. Several notions such as the signature, the index or the discriminant algebra of an involution have been designed by members of the team in collaboration with other teams involved in the project, namely the Dublin, Besançon and Regensburg teams. On the other hand, function fields of quadrics have been investigated in Ghent in collaboration with the Dublin team, leading to several new notions related to the splitting behavior of quadratic forms over function fields of conics. The proposed project is based on expertise gained from this research and is a natural extension of ongoing work.

Linkages: Dublin (Lewis), Besançon (Bayer), Bielefeld (Rehmann, Kersten), Regensburg (Rost) , Minsk (Yanchevskii), St. Petersburg (Merkurjev, Panin). An important book project involves 3 teams: M.-A. Knus, A. S. Merkurjev, M. Rost, J.-P. Tignol, The Book of Involutions, in preparation.

Key scientific staff (man-months in ()): Pascal Mammone (Mons) (12), Jean-Pierre Tignol (Louvain-la-Neuve) (12), Jan Van Geel (Ghent) (12).

Publications:

– A.S. Merkurjev, J.-P. Tignol, The multipliers of similitudes and the Brauer group of homogeneous varieties, J. reine angew. Math. 461, (1995), 13-47.

– E. Bayer-Fluckiger, D.B. Shapiro, J.-P. Tignol, Hyperbolic involutions, Math. Zeit. 214 (1993), 461-476.

6.7 Genova: *Description of recent research activities of the team:* The research activities include algebraic K-theory and its applications to: 1)algebraic geometry, 2) algebraic groups, 3) commutative algebra. and computer algebra.

1) Applications to the study of Chow groups and to the intersection theory on singular varieties. Finiteness properties of the group of cycles on a variety over a number field or a local field. Bloch's conjecture for surfaces and applications to the study of surfaces of general type. Applications of K-theory to study the conductor of curves with singularities that generalize ordinary singularities. Application of K-Theory to the study of topological invariants of real algebraic varieties, in particular in the singular case.

2) Algebraic Groups. Normal subgroup problem of Chevalley groups over rings and its relation with algebraic K-Theory. Representations of rational subgroups of algebraic groups over rings. (2,3) generations of simple groups. Irreducible representations of Chevalley groups. Intermediate subgroups of Chevalley Groups.

3) The applications of K-theory to commutative algebra and to Computer algebra will include the following topics: projective modules over graded algebras, Hilbert functions, generators of ideals of zero dimensional schemes. Computations of the finite groups $SK_1(X)/2$ for a real curve X and $SK_0(X)/2$ for a real surface X. Computer algorithms will be developed in the framework of MACAULAY and COCOA.

Linkages: Paris (Karoubi, Kahn), Regensburg (Rost, Scheiderer), Bielefeld (Kersten, Rehmann), Dublin (Lewis).

Key scientific staff (man-months in ()): Claudio Pedrini (15), Alberto Perelli (15), Luca Barbieri-Viale (15), Giuseppe Valla (15), Maria Evelina Rossi (15), Michele Lattarulo (Universitá di Genova) (15), Ferruccio Orecchia (Universitá di Napoli) (15), Lino Di Martino (15), M.Chiara Tamburini (Universitá di Milano) Yu Chen (Universitá di Torino)

Publications:

– L. Di Martino , N. Vavilov : (2,3)-generation of SL(n,(2,q)) I and II; Communications in Algebra 22 (4), 1321-1347 (1994) and 24 (2) 487-515 (1996)

– L. Barbieri-Viale C. Pedrini and C. Weibel: Roitmann's Theorem for singular Complex surfaces ; Duke J. of Math Vol. 84,No 1 (1996) 155-190

6.8 Dublin: *Description of recent research activities of the team:* 1. Research proposals The study of trace forms and their generalizations on central simple algebras. The study of quadratic- form-theoretic invariants for involutions on central simple algebras. (these topics will have some overlap with work of the Belgian team and the Besançon team) The applications of algebraic K-theory and algebraic L-theory to the computation of knot cobordism groups.
Linkages: Louvain-la-Neuve (Tignol, v. Geel), Besançon (Bayer, Hoffmann), Genova (Pedrini).
Key scientific staff (man-months in ()): D. Lewis (local coordinator) (10), R.Gow (10), K.Hutchinson (10), T.Laffey (10) (all Dublin) and A. Ranicki (12) (Edinburgh).
Publications:
– A. Ranicki (with B. Hughes), "Ends of complexes", Cambridge Tracts 123, Cambridge University Press 1996.
– D.W. Lewis, "Scaled trace forms of central simple algebras", Bull. Belgian Math. Soc. Simon Stevin, 3 (1996), 281-294.

6.9 Odense: *Description of recent research activities of the team:* Team 9 (Odense) came into the research field K-theory mainly from the areas of High Dimensional Geometry and Algebraic Topology.
Bökstedt, Dundas, Madsen, and Rognes work on the computation of K-theory (of rings or spaces) by homotopy theoretic means. In its simplest form, K-theory arises from the study of matrices, and the trace of a matrix is an important invariant in the classical situation. It has been vastly generalized (by Bökstedt, Hsiang and Madsen), viz. to a map $trc : K(A) \to TC(A)$ called the cyclotomic trace.
Here $TC(A)$ is a functor of a strongly homotopy theoretic flavor. It is built from Bökstedt's topological version of Hochschild homology ($TH(A)$) by a construction involving homotopy limits of certain homotopy fixed point sets in $TH(A)$.
This cyclotomic trace has shown its importance in several ways: Hsiang, Bökstedt and Madsen invented it in order to solve the K-theory analogue of Novikov's conjecture about homotopy invariance of higher signatures for manifolds. Later, Bökstedt, Madsen and Rognes have used it to calculate the K-theory of unramified, local number fields.
Dupont studies K-theory in relation to homology of Lie groups considered as discrete groups. This is related to, on the one hand the Scissors Congruence Problem for polytopes in classical geometries (Euclidean, hyperbolic or spherical), on the other hand, the study of characteristic classes for flat bundles.
Pedersen and Munkholm have contributed to the development of controlled K-theory and its applications to problems of a geometric origin, especially the Novikov Conjecture.
Jahren's area is pseudo isotopy theory (and thus A-theory) and its relation to the study of diffeomorphisms of manifolds.
Linkages: Bielefeld (Vogt, Waldhausen,Klein), Strasbourg (Loday), Paris (Karoubi), Dublin/Edinburgh (Ranicki). A workshop on "Controlled Topology, Geometry and Algebra", held Nov. 21 -27, 1994, at Odense University had participants (including graduate students) from Aarhus, Bonn, Bielefeld, Edinburgh, Odense, Genova (and several universities outside Europe).

Key scientific staff (man-months in ()): Hans J. Munkholm (local coordinator) (14). The team includes scientists from Aarhus (DK), Odense (DK), and Oslo (NO): Marcel Bökstedt (14), Bjørn Dundas (14), Johan Dupont (14), Bjørn Jahren (14), Ib Madsen (14), Hans J. Munkholm (14), Erik Kjær Pedersen (14), John Rognes (14).
Publications:
M. Bökstedt, G. Carlsson, R. Cohen, T. Goodwillie, W.C. Hsiang, and *I. Madsen*, "On the algebraic K-theory of simply connected spaces," Duke Math. J. **84** (1996), 541 – 563.
G. Carlsson and *E. K. Pedersen*, "Controlled algebra and the Novikov conjectures for K- and L-theory," Topology **34** (1995), 731 – 758.

6.10 Minsk: *Description of recent research activities of the team:* The team works on properties of the Brauer group over function fields of local curves with arbitrary (good or bad) reduction. Also they work on questions about semisimple algebraic groups, in particular about exceptional groups, but also about R-equivalence and rationality questions and on local-global questions concerning certain arithmetic subgroups over number fields. One of the recent results was the determination of the groups of R-equivalences for special unitary groups (Chernousov/Merkurjev).
Linkages: Bielefeld (Rehmann), Besançon (Bayer), Louvaib-la-Neuve (Tignol, v. Geel), St. Petersburg (Suslin, Merkurjev, Panin).
Key scientific staff (man-months in ()): V. Yanchevskii (12) (local coordinator), V. Chernousov (12), G. Margolin (12), V.I. Guletskii (12).
Publications:
– Guletskii, V.I., Yanchevskii, V.I., Ramification and reciprocity laws in Brauer groups of function fields of number curves, Algebra and Analysis (St. Petersburg Math. Journal) v.5, (1996), 210–228.
– Rapinchuk, A. S., Benyash-Krivetz, V. V., Chernousov, V. I., Representation varieties of the fundamental groups of compact orientable surfaces. Israel Journal of Math. 93 (1996), 29-71.
Particular benefits to the European Community: The team has expertises which are not found among the Western participants, and which therefore are of additional value to the project.
Yanchevskii and his team are leading experts on questions about Brauer groups over local curves, Chernousov has valuable knowledge about rationality questions and about exceptional groups.

6.11 Tbilisi: *Description of recent research activities of the team:* Investigation of the topological Hochschild homology (THH) for various classes of rings. One of the approaches to THH is via the polynomial functors. Recently Friedlander and Suslin used polynomial functors in theory of cohomology of algebraic groups. The team intends to generalize polynomial functors and give applications in various directions. Quillen K-theory for normed algebras is introduced. Using toric varieties will allow to study polyhedral linear groups, for unimodular lattice simplices there is precisely the notion of the general linear group, while new classes of linear groups are obtained for each lattice polyhedron. These groups are described in terms of certain fine combinatorial invariants of the lattice polyhedra. Cohomological investigation of Lie and associative algebras as various of its generalization (Leibniz or non-abelian) will be done.

Linkages: Bielefeld (Vogt, Waldhausen, Rehmann), Odense/Aarhus (Bökstedt, Madsen), Strasbourg (Loday), Paris (Karoubi).
Key scientific staff (man-months in ()): T. Pirashvili (12) (local coordinator), T. Datuashvili (12), J. Gubeladze (12), H. Inassaridze (12), M. Jibladze (12), V. Lomadze (12), (12), D. Pataraia (12), R. Qurdiani (12).
Publications:
– T.Pirashvili. Kan extension and stable homology of Eilenberg-Mac Lane spaces. Topology. 35 (1996), 883-886.
– J.-L. Loday. & T.Pirashvili. Leibniz representations of Lie algebras. J. Algebra 181, (1996), 414-425.
Particular benefits to the European Community: The Tbilisi group is of high value to this project, as it has knowledges and expertises in many fields of K-theory and related things which are important for this project, for example in the areas of topological Hochschild homology, and in cohomological properties of Lie algebras, MacLane homology, stable K-theory, Leibniz algebras and their enveloping algebras.

6.12 St. Petersburg: *Description of recent research activities of the team:* The team works very successfully in many fields of K-theory, in the theory of algebraic groups and in the field of quadratic forms. This is documented for example in the following:
A. Suslin is the editor of
– Algebraic K-theory, Advances in Soviet Math., vol. 4 (1991)
and A. Merkurjev is a coeditor of
– Mathematics in St. Petersburg, Am. Math. Soc. Translations Ser. 2, vol. 174.
In particular, the work of Merkurjev and Panin on index reduction (see below) qualifies them for cooperation in this project.
Linkages: Bielefeld (Kersten, Rehmann), Regensburg (Scheiderer, Rost), Besançon (Bayer, Hoffmann), Louvain-la-Neuve (Tignol, v. Geel), Paris (Kahn, Karoubi), Minsk (Yanchevskii).
An important book project involves 3 teams: M.-A. Knus, A. S. Merkurjev, M. Rost, J.-P. Tignol, The Book of Involutions, in preparation.
Key scientific staff (man-months in ()): A. Suslin (12) (local coordinator), A. Merkurjev (12), I. Panin (12), O. Izhboldin (12), S. Yagunov (12).
Publications:
– A.S. Merkurjev, I.A. Panin and A.R. Wadsworth, Index Reduction Formulas For Twisted Flag Varieties: I, K-Theory, vol.10, (1996), No.6, 517-596.
– V.Voevodskii, A.A.Suslin, Singular homology of abstract algebraic variety, Invent. Math. 23 (1996) 61–94.
Particular benefits to the European Community: A. Suslin and A. Merkurjev are among the world's leading scientists in algebraic K-theory, the theory of algebraic groups over arbitrary fields, and in many related areas. They have achieved many important results which are considered as milestones by the experts. Their St. Petersburg school of young mathematicians like Panin, Izhboldin have also achieved remarkable results in the areas important for this project and are very well known mathematicians in their fields. To include them into this project would be an considerable benefit to it, since they already have a vivid cooperation with most scientists involved into this project.

7. Collaboration between the Teams

The day to day collaboration between the teams will be done on the basis of electronic mail. The workplan contains several conferences and workshops, which will give the members of the projects occasions for the oral exchange of results and ideas.
Since almost all participants already have some meaningful cooperation with each other, based on joint publications, earlier projects of different provenience and several conferences on topics concerning the main theme of this project, there is no doubt that the teams will have a meaningful, interactive and mutually beneficial working level cooperation.
The Dublin team was not involved, as a team, in one of the earlier projects, however, it will contribute to the project by organizing an important conference on quadratic forms, and it will be supported by the help of Andrew Ranicki, who has run a couple of Oberwolfach conferences, and who was a local coordinator in the preceding Science project. This guarantees that this team will properly be integrated into the whole project.
The three partners on project-by-project base also have already team work expertises from a successfully finished INTAS project [IN] mentioned under 8. It therefore is already established that also with these partners a meaningful interactive and mutually beneficial working level cooperation can be maintained.

8. Organization and Management of the Network

Among the participating teams there are collective network expertises by the following joint projects:

[SC] SC1∗-CT91-0756, Title: *K-théorie et ses applications en géométrie, en arithmétique, en topologie et en analyse*, coordinated by M. Karoubi, Paris, during the period March 1992 – August 1995.

[EC] ERB4050PL934139, Euroconference series on *Formes quadratiques, algèbre à division et groupes algébriques linéaires - Relations avec la géométrie algébrique et la K-theorie*, coordinated by E. Bayer, Besançon, Period 1994 – 1996,

[IN] INTAS-93-2618 Research project on *Linear algebraic groups, algebraic K-theory and related structures*, coordinated by U. Rehmann, Bielefeld, Period June 1995 – November 1996.

The network will be coordinated by U. Rehmann Bielefeld. He was the local coordinator for Germany in [SC] and in [EC], and he was the coordinator of [IN]. He also has organized 5 international Oberwolfach conferences on algebraic K-theory, 2 international workshops on K-theory and algebraic groups and, jointly with I. Kersten and A. Merkurjev, one international conference about "Linear algebraic groups and related structures" at the Sonderforschungbereich 343 "Diskrete Strukturen in der Mathematik" at Bielefeld. He moreover is the technical managing editor of the Journal Documenta Mathematica which is produced electronically and in print and is the journal of the German Mathematical Society (DMV).
The local coordinators are C. Scheiderer (Regensburg, Team 2), E. Bayer (Besançon, Team 3), M. Karoubi (Paris, Team 4), J.-L. Loday (Strasbourg, Team 5), J.-P. Tignol, (Louvain-La-Neuve, Team 6), C. Pedrini (Genova, Team 7), D. Lewis (Dublin, Team 8) H.-J. Munkholm (Odense, Team 9) as well as in the teams from Third Countries: V. Yanchevskii (Minsk, Team 10), T. Pirashvili (Tbilisi, Team 11), A. Suslin (St. Petersburg, Team 12).
Almost all coordinators of the EC teams have extended experience in running international scientific conferences and in coordinating networks, and all teams but one (Team 8) have cooperated in at least one of the projects [SC], [EC] or [IN].
The typical technique to coordinate the network will be by electronic means:
All coordinators and most individual participants have access to electronic mail, this will be the typical tool of communication providing good communication between the research teams. Among the coordinators, the typical response time by electronic mail on the internet is less than 1/2 day. (This includes the partners from the Third Countries, as experience from [IN] has shown.) All this, together with the experiences of the administration at Bielefeld and the other universities involved, will guarantee a professional scientific and financial management.
In order to quickly distribute research results, a World Wide Web preprint server will be set up by the network coordinator at U. Bielefeld. This will, besides electronic mail, be used to disseminate informations about important events in the project like seminars, workshops, conferences, and will provide an appropriate tool for monitoring and reporting progress.
Also, several conferences and workshops will be held in the framework of the network, see "10. Training Programme" below.

9. Training Need

The fruitful development of the areas of mathematics covered by this project has resulted in a considerable number of scientists at post-graduate level. Resources for training of young researchers are actually very limited, even on larger universities. This holds in particular for post-doctoral training.
As indicated in section 8, there were already concerted actions and collaborations supported by the EU Science and Euroconferences programmes among the teams of this project in order improve the training possibilities, however, since the number of well qualified young researcher increases, more funding is needed.
Because of the mutual dependencies of the work carried out at the various teams, it is of vital interest for the research in the fields involved in this project that postdoc scientists visit foreign teams for 12-18 months in order to adapt research techniques from other teams and from fields related to but different from their own.
Typically, considerable research advances are made by bringing together experiences of different working groups. For example, it is desirable for a young scientist from a team emphasizing algebraic methods to visit a place for training which is strong in topology, and so on.
Recent progress on, say, the algebraic method of generic splitting would not have been possible without detailed insight into index reduction results, which in turn were based on deep K-theoretic results, depending on topological methods.
These exchanges can be carried out efficiently only on the Community level, as the various national bases are too small, and as the typical national funds are neither suited nor sufficient for these purposes.

10. Training Programme

This project offers an exchange program consisting of 294 post-doctoral fellowships, which shall be supported by additional funds for the organization of *accompanying conferences, summer schools and workshops*, and for mutual short term visits. Every team of this network would be enabled to receive a young scientist from another European country.

All network teams are guided by experienced senior scientists, who are among the leading in there fields, working in well established groups, usually with many scientific guests, in mathematics departments with elaborated training programmes including *lectures, seminars, workshops*, on the doctoral and post-doctoral level. Hence visiting post-doctorals and other staff working on the project are guaranteed to receive optimal training in an academic environment.

The organization of the workshops and summer school of this project will, for example, be based on experiences from [SC], section 8. E.g., three distinct " Summer Schools" were organized in Italy in the framework of [SC] during July 4-10, 1993, July 17-22, 1994 and June 18-23, 1995 by some of the local coordinators of this project.

	Teams:	man-months of training	speciality
1.	Bielefeld	30	Algebra/Topology
2.	Regensburg	30	Algebra
3.	Besançon	30	Algebra
4.	Paris	36	Algebra/Topology
5.	Strasbourg	36	Algebra/Topology
6.	Louvain-laNeuve	30	Algebra
7.	Genova	30	Algebra
8.	Dublin	30	Algebra
9.	Odense	30	Algebra/Topology
	Total :	282	

Special measures to be taken: All teams are – as university departments of mathematics – experienced in offering high level research seminars on various fields of mathematics, which is their usual standard.

Moreover, *dedicated conferences, summer schools and workshops* will be organized by various teams, which will take advantage of the international character of the network by inviting leading experts from other teams.

11. Connections to Industry

No connections to industry are given, since this project is research in pure mathematics.

12. Financial Information

	Teams:	(A)	(B)	(C)	(D)	Total
1.	Bielefeld	112.5	40.0	0	30.5	183
2.	Regensburg	112.5	20.0	0	26.5	159
3.	Besançon	95.0	20.0	0	23.0	138
4.	Paris	114.0	30.0	0	28.0	172
5.	Strasbourg	114.0	30.0	0	28.0	172
6.	Louvain-laNeuve	95.0	20.0	0	23.0	138
7.	Genova	96.0	57.5	0	29.5	183
8.	Dublin	96.0	43.0	0	27.0	166
9.	Odense	112.5	20.0	0	26.5	159
	Total (EC):	947.5	280.5	0	242.0	1470
10.	Minsk					30
11.	Tbilisi					30
12.	St. Petersburg					30
	Total (Third Countries):					90

The amount given in (A) will be used for postdoc employment. The amount in (B) will be used for short term visits, and in order to organize local special training measures as described in section 8. In particular, team 7 requires a higher (B) level because of a planned summer school.