The Components of Lusztig's Nilpotent Variety

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Let Q be a quiver without loops. We denote the corresponding Kac-Moody-Lie algebra by $\mathfrak{g}$ and the universal enveloping algebra of its negative part by $\mathcal{U}^-$. Lusztig's has defined a variety $\mathcal{R}(\Pi(Q), d)_0$, also called Lusztig's nilpotent variety, consisting of nilpotent representations of the preprojective algebra $\Pi(Q)$ of Q of dimension vector d. It was shown by Kashiwara and Saito [KS] that the irreducible components of the various $\mathcal{R}(\Pi(Q), d)_0$, where d runs through all dimension vectors of Q, form the crystal basis of $\mathcal{U}^-$.

The principal aim of this talk is to compute the number of irreducible components of $\mathcal{R}(\Pi(Q), d)_0$ using so-called nilpotent class representations (nc-representations) of Q. Informally an nc-representation of Q assigns to the pair Q, d certain nilpotent classes depending on Q and d, so that the generic nc-representations of Q with dimension vector d are in natural bijection with the irreducible components of $\mathcal{R}(\Pi(Q), d)_0$.

Furthermore, we mention certain applications: the number of irreducible components in the intersection of a nilpotent class with the affine space of strictly upper triangular matrices (in GL_n) can be computed using nc-representations (for the classical results we refer to [BM]). Finally we relate $\mathcal{R}(\Pi(Q), d)_0$ for Q affine and d the imaginary root to the exceptional locus in the resolution of the corresponding Kleinian singularity (see [CS]).

References

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