# The $\mathbb{C}$-valued points of the algebra of strongly regular functions of a Kac-Moody group

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In my preceeding abstract I explained, that the algebra of strongly regular functions $\mathbb{C}[G_m]$ of a symmetrizable minimal Kac-Moody group $G_m$ is isomorphic to the coordinate ring $\mathbb{C}[\widehat{G_m}]$ of a monoid $\widehat{G_m}$. The monoid $\widehat{G_m}$ embeds in $\operatorname{Specm}\mathbb{C}[G_m]$. But in the non-classical case, this map also isn't surjective.

The following parts of $\operatorname{Specm}\mathbb{C}[G_m]$, different from the image of $\widehat{G_m}$, had been already investigated:
Kashiwara used for his construction of the flag variety an open subscheme of the full spectrum, which has a countable covering by big cells.
To define and investigate certain measures, which generalize the classical Haar measure, Pickrell constructed an infinite dimensional complex manifold. To define the manifold structure, he used a countable covering by big cells. This space corresponds to the $\mathbb{C}$-valued points of the subscheme of Kashiwara.
Pickrell also showed, that in the affine case, many interesting completions of $G_m$, corresponding to loop groups, are embedded in this set. Also certain functional analytical closures, for example the closure used by Peterson for his $KAK$-decomposition, embed in the full set $\operatorname{Specm}\mathbb{C}[G_m]$.
A full account with references can be found in the introduction of [M 3].

$\operatorname{Specm}\mathbb{C}[G_m]$ has been determined and investigated in [M 3], [M 4]. Some of the following results have been motivated by results, which are known for reductive algebraic monoids, due to the work of E. Pennell, M. Putcha, and L. Renner. For a full account with references, please look at the introductions of [M 3], [M 4]. Other results have simply been found by analogy with the rank stratification of the monoid of $(n \times n)$-matrices. But the proofs are quite different.

• $\operatorname{Specm}\mathbb{C}[G_m]$ as a $\widehat{G_f} \times \widehat{G_f}$-set:
$\widehat{G_f} \times \widehat{G_f}$ acts in a natural way on $\operatorname{Specm}\mathbb{C}[G_m]$ from the right. The Chevalley involution of $G_m$ extends to an involution $*$ of $\widehat{G_m}$. Factor $\widehat{G_f} \times \widehat{G_f}$ by the $\widehat{G_f} \times \widehat{G_f}$-equivariant equivalence relation, which is generated by $(x, zy) \sim (z^*x, y)$, $x, y \in \widehat{G_f}$, $z \in \widehat{G_m}$. There exists a natural $\widehat{G_f} \times \widehat{G_f}$-equivariant bijection from this quotient set to $\operatorname{Specm}\mathbb{C}[G_m]$.

• The $B_f \times B_f$-orbits, ($B_f$ the formal Borel subgroup of $G_f$):
The Weyl monoid $\widehat{\mathcal{W}}$ determines a system of representatives for the $B_f \times B_f$-orbits, (Birkhoff decomposition). Every orbit is irreducible and principal open in its closure. The closure relation has been determined explicitely. It gives an order on the Weyl monoid, which extends the Bruhat order on the Weyl group.

• The $G_f \times G_f$-orbits:
A system of representatives for the $G_f \times G_f$-orbits has been determined. There are only finitely many orbits. Every orbit is irreducible, and open in its closure. The closure relation of the orbits has been determined.
The biggest orbit coincides with the $\mathbb{C}$-valued points of the open subscheme of Kashiwara, and corresponds to the infinite dimensional complex manifold constructed by Pickrell.

• The big cells of the $G_f \times G_f$-orbits:
There exist big cells of the $G_f \times G_f$-orbits. They have been described. Every big cell of a orbit is principal open and dense in the closure of the orbit. Every orbit can be covered by countably many big cells.

• Transversal stratified slices to the $G_f \times G_f$-orbits have been given at any of their points.

## References


[K,P] V. G. Kac, D. H. Peterson, Regular functions on certain infinite-dimensional groups, in Arithmetic and Geometry, Progress in Math. **36**, Birkhäuser, Boston, 1983, 141-166

[M 3] C. Mokler, The $\mathbb{F}$-valued points of the algebra of strongly regular functions of a Kac-Moody group, accepted by Transformation Groups.

[M 4] C. Mokler, Extending the Bruhat order and the length function from the Weyl group to the Weyl monoid, in preparation.