

## The Brauer-Thrall conjectures.

Abstract. The Brauer-Thrall conjectures were formulated roughly 70 years ago, they have been a guideline for many investigations in the representation theory of artin algebras. They concern the finite-dimensional representations of a finite-dimensional (associative)  $k$ -algebra, where  $k$  is a field, or more generally, the finite length modules over an artin algebra  $\Lambda$ . Note that the Krull-Remak-Schmidt theorem asserts that a  $\Lambda$ -module of finite length can be written in an essentially unique way as a direct sum of indecomposable modules, thus one is interested in the (possibly infinite) number  $m(t)$  of isomorphism classes of indecomposable  $\Lambda$ -modules of length  $t$ . The aim of the lecture is to provide a survey on the present state of knowledge concerning the Brauer-Thrall conjectures and related questions.

## Representations of directed quivers over the algebra of dual numbers.

Abstract: The representations of a quiver  $Q$  over a field  $k$  (the  $kQ$ -modules, where  $kQ$  is the path algebra of  $Q$  over  $k$ ) have been studied for a long time, and one knows quite well the structure of the module category  $\text{mod } kQ$ . It seems to be worthwhile to consider also representations of  $Q$  over arbitrary finite-dimensional  $k$ -algebras  $A$ . We will draw the attention to the case when  $A = k[\epsilon]$  is the algebra of dual numbers (the factor algebra of the polynomial ring  $k[T]$  in one variable  $T$  modulo the ideal generated by  $T^2$ ), thus to the  $\Lambda$ -modules, where  $\Lambda = kQ[\epsilon] = kQ[T]/\langle T^2 \rangle$ .

The algebra  $\Lambda$  is a 1-Gorenstein algebra, thus the torsionless  $\Lambda$ -modules are known to be of special interest (as the Gorenstein-projective or maximal Cohen-Macaulay modules). They form a Frobenius category  $\mathcal{L}$ , thus the corresponding stable category  $\underline{\mathcal{L}}$  is a triangulated category. The category  $\mathcal{L}$  is the category of perfect differential  $kQ$ -modules and  $\underline{\mathcal{L}}$  is the corresponding homotopy category. The category  $\underline{\mathcal{L}}$  turns out to be triangle equivalent to the orbit category of the derived category  $D^b(\text{mod } kQ)$  modulo the shift and the homology functor  $H: \text{mod } \Lambda \rightarrow \text{mod } kQ$  yields a bijection between the indecomposables in  $\underline{\mathcal{L}}$  and those in  $\text{mod } kQ$ , the inverse of  $H$  is given by taking minimal  $\mathcal{L}$ -approximations. As we will show, the kernel of the restriction of  $H$  to  $\mathcal{L}$  is a finitely generated ideal which can be described explicitly. This is a report on joint work with Pu Zhang.

## Wild algebras.

Abstract. It has been observed 50 years ago by Corner, Brenner and Butler that sufficiently complicated finite-dimensional algebras  $\Lambda$  have some quite wild behavior: any finite-dimensional algebra  $\Lambda'$  can be realized as the endomorphism ring of some  $\Lambda$ -module (at least nearly), or, even better: the category of all  $\Lambda'$ -modules can be realized as a full exact subcategory of the category of all  $\Lambda$ -modules (again, at least nearly). A general tame-wild-dichotomy result was presented by Drozd in 1984, however his definition of wildness deviates from the observations of Corner-Brenner-Butler. As a remedy, there is the notion of controlled wildness, it will be explained in the lecture. A proof of the conjecture that all wild algebras are even controlled wild, has been announced by Drozd in 2007 (but not yet published). In addition, we will present some methods for deciding whether an algebra is wild, and we will discuss some conjectures of Han Yang concerning wild algebras.