Vorlesungsergänzung vom 03.12.2008

$$
\begin{aligned}
exp_a(x+y) &= exp((x+y)ln_a) \\
&= exp(xln_a + yln_a) \\
&= exp(xln_a) \cdot exp(yln_a) \\
&= exp_a(x) \cdot exp_a(y)
\end{aligned}
$$

$$
\begin{aligned}
\forall n \in \mathbb{N} exp_a(n) &= a^n \\
n &= 1 \\
exp_a(1) &= exp(1 \cdot ln_a) \\
&= exp(ln_a) = a
\end{aligned}
$$

$$\boxed{\forall x \in \mathbb{R}_+^- \quad exp(ln_x) = x}$$

$$n \in \mathbb{N} \quad exp_a(n) = a^n$$

$$
\begin{aligned}
n &\to n+1 \quad ? \\
exp_a(n+1) &= a^{n+1} \\
exp_a(n) \cdot exp_a(1) &= a^n \cdot a = a^{n-1} \\
a^n \qquad & a \\
\forall n \in \mathbb{N} \quad exp_a(n) &= a^n
\end{aligned}
$$

$$
\begin{aligned}
n &= 0 \\
exp_a(0) &= exp(0 \cdot ln_a) \\
&= exp(0) = 1 = a^0
\end{aligned}
$$

$$\begin{aligned}
\forall n \in \mathbb{Z}/\mathbb{N}\ \ ?\\
exp_a(-n) &= a^{-n} \quad ,\forall n \in \mathbb{N}\\
exp_a(-n)\cdot exp_a(n) &= exp_a(-n+n)\\
&= exp(0)\\
&= 1\\
exp_a(-n) &= \frac{1}{exp_a(n)}\\
&= \frac{1}{a^n}\\
&= a^{-n}
\end{aligned}$$

$$\forall q \in \mathbb{N} \quad \forall x \in \mathbb{R}$$

$$\begin{aligned}
exp_a(q\cdot x) &= exp_a((q-1)x)\cdot exp_a(x)\\
&= \dots\\
&= \underbrace{exp_a(x)\cdot \ldots \cdot exp_a(x)}_{q \quad mal}\\
&= (exp_a(x))^q
\end{aligned}$$

$$\begin{aligned}
A\gamma \geqslant \eta \quad \eta > \frac{s}{\gamma} > 0\\
\frac{s}{\gamma} = \gamma - \frac{s}{\gamma} < \eta < \gamma + \frac{s}{\gamma}\\
\xi = \frac{s}{\gamma} > 0 \exists \gamma \in \mathbb{N} \quad A\gamma \geqslant \eta\\
|\sigma^n - \sigma| < \xi \Leftrightarrow \gamma - \epsilon < \eta < \gamma + \xi\\
A\xi > 0 \quad \exists n \in \mathbb{N} A\gamma \geqslant n\\
\sigma^\gamma \to \sigma'\gamma \to \infty
\end{aligned}$$

$$\begin{aligned}
\forall x_n \in \mathbb{R}_+^* &\quad \underline{x_n \to \infty},\ n \to \infty\\
\lim_{n\to\infty} ln x_n = \infty\\
\forall c > 0 &\quad \exists N \in \mathbb{N} \qquad \forall n \geq N ln\, x_n > c\\
\forall c_1 = e^{c+1} > 0 &\quad \exists N_0 \leftarrow \mathbb{N} \quad \forall n \geq N_0\\
&\quad \underline{x_n > e^{c+1}}\\
ln\, x_n > ln\, e^c + 1 = (c+1)\cdot 1 > c
\end{aligned}$$

$$
\begin{aligned}
\frac{e^x - 1}{x} &= \frac{\sum\limits_{n=0}^{\infty} \frac{x^n}{n!} - 1}{x} \\
&= \frac{1 = \frac{x^0}{0!} + \sum\limits_{n=1}^{\infty} \frac{x^n}{n!} - 1}{x} \\
&= \frac{x + \sum\limits_{n=1}^{\infty} \frac{x^n}{n!} - x}{x} \\
&= \frac{1}{x} \sum_{n=1}^{\infty} \frac{x^n}{n!} \\
&= \frac{1}{x} (\underbrace{\frac{x^1}{1!}}_{x} + \sum_{n=2}^{\infty} \frac{x^n}{n!}) \\
&= \frac{1}{x} (x + x^2 \sum_{n=2}^{\infty} \frac{x^{n-2}}{n!} \\
&= 1 + x \underline{\sum_{n=2}^{\infty} \frac{x^{n-2}}{n!}}
\end{aligned}
$$

$$
\begin{aligned}
| \sum_{n=2}^{\infty} \frac{x^{n-2}}{n!} | &\leq \sum_{n=2}^{\infty} \frac{|x|^{n-2}}{n!} \\
&\leq \sum_{n=2}^{\infty} \frac{1}{n!} \quad , \quad \forall x : |x| \leq 1
\end{aligned}
$$

$$
\sum_{n=0}^{\infty} \frac{1}{n!} = e^1
$$

$$
\forall x : |x| \leq 1 \quad 1 \leq \frac{e^{x-1}}{x} \leq 1 + x \cdot e
$$

$$
1, \ x \to 0 \quad 1, \ x \to 0 \quad 1, \ x \to 0
$$

$$\frac{1}{2}(z+\bar{z}) = \frac{1}{2}(x+\underbrace{iy}_{0}+x-\underbrace{iy}_{0}) = \frac{2x}{2}$$

$$z = x+iy,\ \bar{z} = x-iy = x = Re(z)$$

$$\frac{1}{2i}(z-\bar{z}) = \frac{1}{2i}(\underbrace{x}_{0}+iy-\underbrace{x}_{0}+iy) = \frac{\overbrace{2i}^{0}\ y}{\underbrace{2i}_{0}}$$

$$= y = I_m(z)$$

$$\bar{\bar{z}} = z$$

$$\bar{z} = x-iy = x+i(-y)$$

$$\bar{\bar{z}} = x-i(-y) = x+iy = z$$

$$z\cdot\bar{z} = x^2+\zeta^2$$

$$(x+iy)(x-iy) = x^2+i\underbrace{y}_{0}x-i\underbrace{x}_{0}y-\underbrace{i^2y^2}_{-1}$$

$$= x^2+y^2$$

$$z_1 = x_1+iy_1 z_2 = x_2+iy_2$$

$$\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$$

$$\overline{x_1+iy_1+x_2+iy_2} = \overline{(x_1+x_2)+i(y_1+y_2)}$$

$$= x_1+x_2-i(y_1+y_2)$$

$$= \underbrace{x_1-iy_1}_{\bar{z_1}}+\underbrace{x_2-iy_2}_{\bar{z_2}}$$

$$\overline{z_1\cdot z_2} = \bar{z_1}\cdot\bar{z_2}$$

$$z_1\cdot z_2 = (x_1+iy_1)(x_2+iy_2)$$

$$= x_1x_2-y_1y_2+i(y_1x_2+x_1y_2)$$

$$\overline{z_1\cdot z_2} = x_1x_2-y_1y_2+i(y_1x_2+x_1y_2)$$

$$\underline{\underline{\bar{z_1}\cdot\bar{z_2}}} = (x_1-iy_1)(x_2-iy_2)$$

$$= x_1x_2-y_1y_2-iy_1x_2-ix_1y_2$$

$$= x_1x_2-y_1y_2-i(y_1x_2+x_1y_2)$$

$$|\bar{z}| = (x^2+(-y)^2)^{\frac{1}{2}}$$

$$= (x^2+y^2)^{\frac{1}{2}}$$

$$= |\bar{z}|$$

$$\frac{1}{\alpha}\lim_{n\to\infty}\frac{\alpha\ ln\ x_n}{exp(\alpha\ ln\ x_n)}$$

$$ln\ x_n \to \infty, \quad x_n \to \infty$$

$$y_n = \alpha\ ln\ x_n \to \infty, \quad n \to \infty$$

$$= \frac{1}{\alpha} \lim_{n \to \infty} \frac{y_n}{exp(y_n)} =$$

$$k = 1 \quad imBeispiel \quad (i)$$

$$= 0$$