

Exercises to Stochastic Analysis

Sheet 6

Total points: 13

Submission before: Friday, 25.11.2022, 12:00 noon

(*[Parts of] Exercises marked with “*” are additional exercises.*)

Problem 1 (Pathwise quadratic variation vs. quadratic variation process). (6 Points)

It is a crucial step in this lecture to understand the transition from the pathwise Itô-calculus to the level of processes and you should particularly try to understand precisely how the pathwise quadratic variation and the notion of quadratic variation process are related. This exercise shall help you to take a look back at both concepts and make a comparison.

For simplicity, you can restrict your answers to one-dimensional processes.

- (a) For which stochastic processes do we know the pathwise quadratic variation of Ch.1 exists, and what do we know about it in these cases? Note that we just *assumed* the existence of the qu. var. several times in Ch.1! For which class of stochastic processes do we have a quadratic variation process according to Ch.2.2?
- (b) How exactly are the pathwise notion from Ch.1 and the quadratic variation process of Ch.2.2 related? For your answer, have a look at Rem. 2.2.9 (ii)+(iii). Looking back at (a), does this imply the existence of the pathwise quadratic variation for processes for which we could only assume this existence in Ch.1?
- (c) What do you know about (i) uniqueness and (ii) independence of the chosen sequence of partitions for the pathwise quadratic variation and the quadratic covariation process, respectively?
- (d) Compare the pathwise covariation from Ch.1 with the covariation process from Ch.2.2 in terms of definition, properties and how they are related.

Problem 2 (Polarization identity for quadratic covariation process). (3 Points)

We know that the pathwise covariation $\langle X, Y \rangle$ from Ch.1 satisfies the polarization identity (if it is well-defined). Here we prove the intuitively reasonable claim that this is also true for the covariation process from Ch.2.2.

Prove Remark 2.2.14.

You should carefully distinguish the pathwise and the process level!

Problem 3 (Kunita-Watanabe inequality). (4 Points)

Prove Lemma 2.2.16., including the “in particular”-part of the assertion.