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Dear colleagues,

we would like to invite you to attend the

Summer School and Workshop on
Springer correspondence and applications

which are scheduled for September 21 to September 28, 1998, in Picquigny, Picardie, France. This village is located half way between Paris and Calais at the river Somme.

During the summer school we will basically read the article of Shoji in Astérisque 168. The reading of this article will occupy Tuesday to Thursday or Friday and then Friday to Sunday morning we will invite distinguished experts in the field to inform in small courses about more recent developments in the field.

For the first part we will learn about the very first steps on material like the general theory of algebraic groups, étale cohomlogy and perverse sheaves in the first lectures, for which there are very readable introductions available. The succeeding talks will be on Shoji's paper itself, followed by his algorithm to construct Green functions as it is described in the Aracata 1986 Conference Proceedings. Lusztig developed out of that a strategy, using character sheaves, to compute the whole character table

The summer school is intended to follow the tradition of previous summer schools in representation theory. The idea is that every participant chooses one subject as listed below for subject of a short talk during the first part of the school. Every participant is supposed to give a short talk. The typical participant will not be an expert on algebraic groups, but she or he will be working in algebra and be interested in jointly learning a new subject. There are no special prerequisites (apart from standard knowledge on groups, rings and homological algebra and a little bit of algebraic geometry), but the participants should be willing to prepare themselves and their talks in advance. Experience shows that it is recommendable to work in groups. Especially one may and should subdivide the talks listed below into smaller parts.

The topic of the summer school might be interesting for people working in representation theory, group theory, combinatorics, algebraic geometry, or other areas. We would like to especially invite colleagues who either are working for a PhD or recently got one.

We are applying for financial support from the European Union. In case of a positive decision, we will be able to contribute substantially to the costs of accommodation and travel expenses. Since we already got a funding by the GDR "groupes réductifs" and the Université de Picardie, we are at least able to cover parts of the local expenses. The number of participants is limited to about 25 persons.

Below you find some details about the program of the summer school and of the workshop. We suggest that you choose a section you would like to talk about. The decision of the European Union is expected at latest in April. It would be nice, if you could inform us until end of April 1998, (preferably by e–mail to Alexander: alex@math.jussieu.fr or Alexander.Zimmermann@u-picardie.fr) that you intend to come and which section you would like to talk about. As soon as we know about the financial support and the potential participants we will send you more details, including a final program.

We hope to hear from you and to see you next September in Picquigny,

for the organizers, Alexander Zimmermann

Here are some informations on the subject of the summer school.

1 Whom to contact

We are four organizers: François Digne, Meinolf Geck, Jean Michel and Alexander Zimmermann. For any questions concerning the summer school and the talks please contact Alexander Zimmermann (alex@math.jussieu.fr or az-math@u-picardie.fr).

2 Description

The theory of character sheaves, developed by G. Lusztig in a series of papers [13, 14], was invented in order to attack the problem of computing the character tables of finite groups of Lie type. It builds on earlier work of Lusztig's on the classification of irreducible characters [12], and uses the theory of perverse sheaves, in the algebraic version due to Beilinson, Bernstein, Deligne and Gabber [1].

The flavour is to define geometrical objects, the perverse sheaves, on which an algebraic group acts. This defines class functions of this group. These class functions can be determined using an abstract construction, the Springer correspondence. There is an injective map from the irreducible characters of the Weyl group W of the connected algebraic reductive group G into a set of pairs $(C, \mathcal{E})$ where C is a unipotent conjugacy class, and $\mathcal{E}$ is a so-called 'G-equivariant irreducible local system on C'; one of these geometric objects. Using a geometric realization of so-called 'Green functions' via character sheaves and the Borho-MacPherson theorem [2] this permits to get an *algorithm* to compute the Green functions, which in turn can be transformed by some Fourier transform into characters of G. Not all pairs $(C, \mathcal{E})$ occur as Springer correspondents. The generalized Springer correspondence, due to Lusztig, produces the missing pairs. One can then see that all characters are produced. This algorithm is implemented in the computer packages GAP resp. CHEVIE.

3 Tentative program

We shall follow in the first part of the summer school Shoji's description of the Springer correspondence in [19] together with a brief review on derived categories, perverse sheaves, ℓ-adic topology and generalities on algebraic groups (following [1, 27, 3, 26]). The aim is to cover sufficient material in order to understand the Borho-MacPherson theorem [2]. Then, we shall describe the applications to the computation of Green functions, as in [18]. This leads to an explicit and practical algorithm, for which we give all the details in the smallest non-trivial case, groups of type G_2. As time permits, we will then go with the generalized Springer correspondence.

In the second half we will invite distinguished specialists of the field to give more advanced results in this direction. Tony Springer already accepted to give a lecture.

4 The place where it happens

The whole summer school will take place in the "Abbaye Notre Dame du Gard" in the valley of the river Somme. This monastery was founded in the 12th century and it remained until the french revolution. After the Napoleon wars it was reoccupied by monks until middle of last century when the new constructed railway disturbed this quiet place. In the 1960s the "Frères Auxiliaires" of the nearby monastery St Riquier reconstructed the "corps de logis" which dates from the 18th century. The building is at once the working place of the monks and from time to time the place for seminars and small congresses in the building, hosted by the "Frères Auxiliaires". The ruins of the gothic church are still visible.

The river Somme is quite long, several hundred kilometers, but it originates near St Quentin at an altitude of about 100 meters and goes into the channel. As one can imagine

it gives rise to big moors, lakes and hundreds of sidearms. The whole valley and in particular the delta is famous for is rich variety of bird population. Amiens is nearby with its famous gothic cathedral, the biggest in France. For a virtual visit see
http://www.learn.columbia.edu/Amiens.html or
http://www.u-picardie.fr/~patrick/Cathedrale/visite.html

5 The lectures

1. Variety of Borel subgroups following [19, Theorem 1.2] (60 min)
2. Dimension estimates, [19, §§ 1.3 to 1.4] (45 min)
3. The Grothendieck stratification of G [19, § 2] (45 min)
4. Etale topology and cohomology following [26] (60 + 45 min)
5. Generalities on derived categories and perverse sheaves following [19, § 3] or [1] (30 + 60 min)
6. Springer representations [19, § 4] (45 min)
7. Weyl group action on ℓ-adic cohomology [19, § 5] (45 min)
8. Borho-MacPherson's theorem and Springer correspondence I and II [19, § 6] (60 + 60 min)
9. The algorithm for constructing Green functions [18, § 5] (60 min)
10. Cuspidal pairs [19, § 7] (60 min)
11. Admissible complexes [19, § 8] (45 min)
12. Statement of the generalized Springer correspondence [19, § 9] (45 min)
13. The algorithm to determine the generalized Springer correspondence for type G_2 [22] (45 min)

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