Optimization and Dynamics
Summer semester 2015

Exercise sheet 10

Due 12pm, 19.06.2015

1. Determine whether or not each of the following functions fulfils a Lipschitz condition on the set $\Omega := [0,1] \times [0,1]$.

   (a) $f(t,x) = t^2 + x^2$

   (b) $f(t,x) = \sin t \cdot \cos t$

   (c) $f(t,x) = |t - x|$

2. Consider the initial value problem $\begin{cases} x' = tx \\ x(0) = x_0 \,. \end{cases}$

   (a) Does the function $f(t,x) = tx$ fulfil a Lipschitz condition on $\mathbb{R} \times \mathbb{R}$?

   (b) Solve the differential equation. Is there a solution for all $x_0 \in \mathbb{R}$?

   (c) Is the solution in each case unique?

3. Consider the initial value problem $\begin{cases} x' = tx^2 \\ x(0) = x_0 \,. \end{cases}$

   (a) Does the function $f(t,x) = tx^2$ fulfil a Lipschitz condition on $\mathbb{R} \times \mathbb{R}$?

   (b) Solve the differential equation. Is there a solution for all $x_0 \in \mathbb{R}$? Is it defined on $\mathbb{R}$ or just on a subset of $\mathbb{R}$?

   (c) Is the solution in each case unique?

4. Consider the initial value problem $\begin{cases} x' = \sqrt{x} \\ x(0) = x_0 \,. \end{cases}$

   (a) Does the function $f(x) = \sqrt{x}$ fulfil a Lipschitz condition on $\mathbb{R}^+$?

   (b) Solve the differential equation. Is there a solution for all $x_0 \in \mathbb{R}^+$?

   (c) Is the solution in each case unique?