Optimization and Dynamics
Summer semester 2015

Exercise sheet 11

Due 12pm, 26.06.2015

1. Let A and B be commuting $n \times n$ matrices and S an invertible $n \times n$ matrix. Prove the following identities.

   (a) $e^{A+B} = e^A e^B$

   (b) $(e^A)^{-1} = e^{-A}$

   (c) $(e^A)^m = e^{mA}$, $m \in \mathbb{Z}$

   (d) $e^{SAS^{-1}} = Se^A S^{-1}$

2. Let $A \neq 0$ be a nilpotent matrix.

   (a) Show that A is not invertible.

   (b) Show that $A + I$ is invertible.

   (c) What can you say about the eigenvalues of A?

3. Let P be a projection matrix, that is, a matrix such that $P^2 = P$. Show that

$$e^P = I + (e-1)P.$$

4. Let $A = \begin{pmatrix} -1 & 2 \\ -4 & 5 \end{pmatrix}$.

   (a) Write down the characteristic equation of A, and show that A fulfils it.

   (b) Diagonalise A and hence find A^{12} and e^A.