

## Optimization and Dynamics

Summer semester 2015

### Exercise sheet 12

Due 12pm, 03.07.2015

1. Let  $A = \begin{pmatrix} -1 & 2 \\ -4 & 5 \end{pmatrix}$ .

- (a) Find the polynomial  $q$  such that  $e^A = q(A)$ .
- (b) Hence find  $e^A$ .

2. Find  $e^{tA}$ , where  $t \in \mathbb{R}$ , for

(a)  $A = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}$  and

(b)  $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ .

3. Let  $A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ .

- (a) Show that  $A^n = (-1)^{n+1}A$  for all  $n \in \mathbb{N}$ .
- (b) Hence calculate  $e^{tA}$ .

4. Let  $A = \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix}$

- (a) Find  $e^A$ .

(b) Hence solve the IVP  $x' = Ax$  with  $x(0) = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ .