

Optimization and Dynamics

Summer semester 2015

Exercise sheet 13

Due 12pm, 10.07.2015

1. Discuss the stability of the system $x' = Ax$ in the case $A = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ where $\lambda_1 \neq 0$ and $\lambda_2 = 0$. Sketch phase diagrams.

2. (a) Consider the system

$$\begin{cases} x_1' = 2x_1 - 8x_2 \\ x_2' = x_1 - 4x_2. \end{cases}$$

Find all fixed points and discuss their stability. Write down the general solution to the system and draw a phase diagram.

- (b) Now consider the system

$$\begin{cases} x_1' = 2x_1 + ax_2 \\ x_2' = x_1 - 4x_2. \end{cases}$$

For which values of $a \in \mathbb{R}$ is the fixed point $x = 0$ asymptotically stable?

3. For each of the following systems, find the fixed points and discuss their stability.

(a)
$$\begin{cases} x' = -3x + 4y \\ y' = -2x + 3y \end{cases}$$

(b)
$$\begin{cases} x' = 4x - 2y \\ y' = 5x + 2y \end{cases}$$

4. Consider the linear differential equation $x' = Ax$ where

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

- (a) Find the general solution.
(b) Find the fixed points and discuss their stability.
(c) Sketch the phase diagram.